



Oxford Cambridge and RSA

Wednesday 5 June 2019 – Morning

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Time allowed: 2 hours



You must have:

- Printed Answer Booklet

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g \text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets [].
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \frac{1}{2}n(a + l) = \frac{1}{2}n\{2a + (n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

Differentiation from first principles

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

Small angle approximations

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{1}{2}\theta^2, \tan \theta \approx \theta \text{ where } \theta \text{ is measured in radians}$$

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left(A \pm B \neq \left(k + \frac{1}{2}\right)\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}$, where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x) = 0$: $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

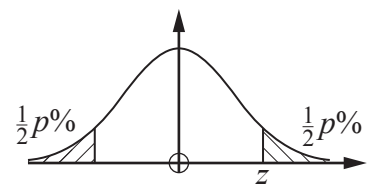
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$ and $\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576

**Kinematics**

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

Section A (25 marks)

1 In this question you must show detailed reasoning.

Show that $\int_4^9 (2x + \sqrt{x}) dx = \frac{233}{3}$. [3]

- 2** Show that the line which passes through the points $(2, -4)$ and $(-1, 5)$ does not intersect the line $3x + y = 10$. [3]

- 3** The function $f(x)$ is given by $f(x) = (1 - ax)^{-3}$, where a is a non-zero constant. In the binomial expansion of $f(x)$, the coefficients of x and x^2 are equal.

(a) Find the value of a . [3]

(b) Using this value for a ,

(i) state the set of values of x for which the binomial expansion is valid, [1]

(ii) write down the quadratic function which approximates $f(x)$ when x is small. [1]

- 4** Fig. 4 shows a uniform beam of mass 4 kg and length 2.4 m resting on two supports P and Q. P is at one end of the beam and Q is 0.3 m from the other end. Determine whether a person of mass 50 kg can tip the beam by standing on it. [3]

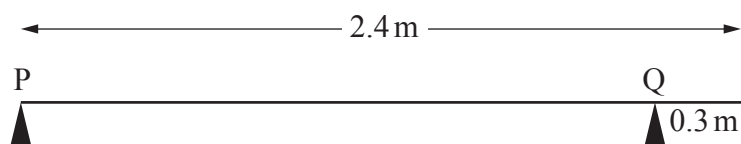


Fig. 4

- 5** A car of mass 1200 kg travels from rest along a straight horizontal road. The driving force is 4000 N and the total of all resistances to motion is 800 N. Calculate the velocity of the car after 9 seconds. [4]

- 6** (a) Prove that $\frac{\sin \theta}{1 - \cos \theta} - \frac{1}{\sin \theta} = \cot \theta$. [4]

(b) Hence find the exact roots of the equation $\frac{\sin \theta}{1 - \cos \theta} - \frac{1}{\sin \theta} = 3 \tan \theta$ in the interval $0 \leq \theta \leq \pi$. [3]

Answer **all** the questions.

Section B (75 marks)

- 7 The velocity $v \text{ ms}^{-1}$ of a particle at time $t \text{ s}$ is given by
 $v = 0.5t(7 - t)$.
 Determine whether the **speed** of the particle is increasing or decreasing when $t = 8$. [4]
- 8 An arithmetic series has first term 9300 and 10th term 3900.
 (a) Show that the 20th term of the series is negative. [3]
 (b) The sum of the first n terms is denoted by S . Find the greatest value of S as n varies. [4]
- 9 A cannonball is fired from a point on horizontal ground at 100 ms^{-1} at an angle of 25° above the horizontal. Ignoring air resistance, calculate
 (a) the greatest height the cannonball reaches, [3]
 (b) the range of the cannonball. [4]
- 10 (a) Express $7 \cos x - 2 \sin x$ in the form $R \cos(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact value of R and the value of α correct to 3 significant figures. [4]
 (b) Give details of a sequence of two transformations which maps the curve $y = \sec x$ onto the curve $y = \frac{1}{7 \cos x - 2 \sin x}$. [3]
- 11 In this question, the unit vector $\mathbf{i}$ is horizontal and the unit vector $\mathbf{j}$ is vertically upwards.
 A particle of mass 0.8 kg moves under the action of its weight and two forces given by $(k\mathbf{i} + 5\mathbf{j}) \text{ N}$ and $(4\mathbf{i} + 3\mathbf{j}) \text{ N}$. The acceleration of the particle is vertically upwards.
 (a) Write down the value of k . [1]
 Initially the velocity of the particle is $(4\mathbf{i} + 7\mathbf{j}) \text{ ms}^{-1}$.
 (b) Find the velocity of the particle 10 seconds later. [4]

- 12** Fig. 12 shows a curve C with parametric equations $x = 4t^2$, $y = 4t$. The point P , with parameter t , is a general point on the curve. Q is the point on the line $x + 4 = 0$ such that PQ is parallel to the x -axis. R is the point $(4, 0)$.

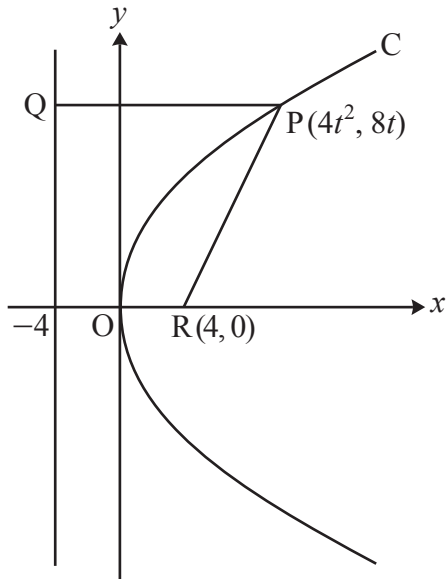


Fig. 12

- (a) Show algebraically that P is equidistant from Q and R . [4]
- (b) Find a cartesian equation of C . [2]
- 13** A 15 kg box is suspended in the air by a rope which makes an angle of 30° with the vertical. The box is held in place by a string which is horizontal.
- (a) Draw a diagram showing the forces acting on the box. [1]
- (b) Calculate the tension in the rope. [2]
- (c) Calculate the tension in the string. [2]

- 14 Fig. 14 shows a circle with centre O and radius r cm. The chord AB is such that angle $AOB = x$ radians. The area of the shaded segment formed by AB is 5% of the area of the circle.

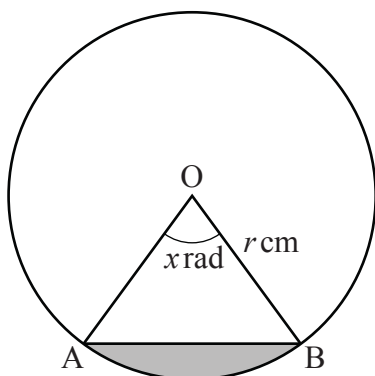


Fig. 14

- (a) Show that $x - \sin x - \frac{1}{10}\pi = 0$. [4]

The Newton-Raphson method is to be used to find x .

- (b) Write down the iterative formula to be used for the equation in part (a). [1]

- (c) Use three iterations of the Newton-Raphson method with $x_0 = 1.2$ to find the value of x to a suitable degree of accuracy. [3]

- 15 A model for the motion of a small object falling through a thick fluid can be expressed using the differential equation

$$\frac{dv}{dt} = 9.8 - kv,$$

where $v \text{ m s}^{-1}$ is the velocity after t s and k is a positive constant.

- (a) Given that $v = 0$ when $t = 0$, solve the differential equation to find v in terms of t and k . [7]

- (b) Sketch the graph of v against t . [2]

Experiments show that for large values of t , the velocity tends to 7 m s^{-1} .

- (c) Find the value of k . [2]

- (d) Find the value of t for which $v = 3.5$. [1]

- 16 A particle of mass 2 kg slides down a plane inclined at 20° to the horizontal. The particle has an initial velocity of 1.4 ms^{-1} down the plane. Two models for the particle's motion are proposed.

In model A the plane is taken to be smooth.

- (a) Calculate the time that model A predicts for the particle to slide the first 0.7 m. [5]
- (b) Explain why model A is likely to underestimate the time taken. [1]

In model B the plane is taken to be rough, with a constant coefficient of friction between the particle and the plane.

- (c) Calculate the acceleration of the particle predicted by model B given that it takes 0.8 s to slide the first 0.7 m. [2]
- (d) Find the coefficient of friction predicted by model B, giving your answer correct to 3 significant figures. [6]

END OF QUESTION PAPER

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