



GCE

Mathematics B (MEI)

H640/02: Pure Mathematics and Statistics

Advanced GCE

Mark Scheme for June 2019

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Text Instructions

Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so.
- When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value.
 - When a value is not given in the paper accept any answer that agrees with the correct value to **2 s.f.**

Follow through should be used so that only one mark is lost for each distinct accuracy error.

- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.
- i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1	(a)		$0.2 + 0.1 + k + 2k + 4k [= 1]$ soi [$k =$] 0.1	M1 A1 [2]	1.1a 1.1		
1	(b)		$1 - 2 \times$ their k 0.8 or $\frac{4}{5}$ cao	M1 A1 [2]	1.1 1.1	$1 -$ their $P(X = 4)$ oe	
2	(a)		$k(x^2 + 5)^{11}$ seen $24x(x^2 + 5)^{11}$	M1 A1 [2]	1.1a 1.1		
2	(b)		$a(x^2 + 5)^{12}$ $2(x^2 + 5)^{12} (+c)$	M1 A1 [2]	1.1 1.1	condone omission of $+ c$	A1 FT their $kx(x^2 + 5)^{11}$ from part (a)
3	(a)		2.8 to 2.81 BC	B1 [1]	1.1		NB 2.8063583815029..

Question			Answer	Marks	AOs		Guidance
3	(b)		eg the data is already grouped oe eg we do not have the original raw data oe eg we are using the mid-point of the intervals eg we are assuming the data are uniformly distributed across each interval oe	B1 [1]	2.4		
3	(c)		eg using upper class limit in each case gives mean is 3.4...so it is possible that mean is more than 3	B1 [1]	2.3	or eg need 520 or more and using the upper limits gives 590	simply stating eg the mean could be 3.4 is insufficient
4	(a)		$\frac{0.5}{2} \left[\sqrt{1+(-1)^3} + 2\sqrt{1+(-0.5)^3} + \sqrt{1+0^3} \right]$ oe $\sqrt{1+(-0.5)^3}$ soi 0.717707 cao	M1 B1 A1 [3]	1.1 1.1 1.1	condone omission of brackets NB $\frac{\sqrt{14}}{4} = 0.9(35414346693)$ NB $\frac{2+\sqrt{14}}{8}$ unsupported implies M1B1	must be three terms in the bracket if unsupported allow SC3 for 0.717707 and SC2 for 0.717707173347 unsupported to 7 or more dp
4	(b)		under-estimate since curve is concave down /convex up oe	B1 [1]	2.4	or eg the slant lines of both trapezia are entirely below the curve allow annotated diagram with at least one trapezium	condone eg trapezium below curve allow integral is 0.841309 BC so my answer is an underestimate

Question			Answer	Marks	AOs	Guidance
5	(a)		2.5	B1 [1]	1.1	
5	(b)		positive skew cao	B1 [1]	1.1	
5	(c)		1	B1 [1]	1.1	
6			$k \frac{x^{-4}}{-4} \ln x - \int k \frac{x^{-4}}{-4} \times \frac{1}{x} dx$ oe $[32] \frac{x^{-4}}{-4} \ln x - \int [32] \frac{x^{-4}}{-4} \times \frac{1}{x} dx$ oe $-8x^{-4} \ln x - 2x^{-4} + c$ oe isw	M1 A1 A1 A1 [4]	3.1a 1.1 1.1 1.1	allow sign errors only all correct two of three elements correct all three elements correct
7	(a)		$\theta = \frac{72.576}{r^2}$ or $72.576 r^{-2}$ isw	B1 [1]	1.1	eg $\frac{9072}{125r^2}$ or $\frac{9072r^{-2}}{125}$
7	(b)		$r\theta + 2r$ or $r(\theta + 2) = 24.48$ seen and $\theta = \frac{24.48 - 2r}{r}$ or equivalent constructive step to give $\frac{24.48}{r} - 2$ AG	B1 [1]	1.1	or $2\pi r \frac{\theta}{2\pi} + 2r = 24.48$

Question			Answer	Marks	AOs		Guidance
7	(c)		$their \frac{2 \times 36.288}{r^2} = \frac{24.48}{r} - 2$ oe $r^2 - 12.24r + 36.288 [= 0]$ $[r =] 5.04$ or 7.2 oe	M1 M1 A1 [3]	3.1a 2.1 1.1	NB 72.576 quadratic obtained in form $f(r) [= 0]$	allow B3 for 5.04 and 7.2 unsupported or allow SC3 for obtaining and solving an equation for θ and then finding both values of r
8	(a)		0.4×0.4 or 0.6×0.2 seen $0.4 \times 0.4 + 0.6 \times 0.2$ 0.28	M1 A1 A1 [3]	3.1a 1.1 1.1	or tree diagram with correct outcomes and probabilities shown	allow $0.4 \times 0.4 \times 0.4$ or $0.4 \times 0.6 \times 0.2$ mark the final answer
8	(b)		$\frac{1}{120}$ or $\frac{5}{120}$ seen $\frac{1}{24}$ or 0.0416666...to 2 or more sf	M1 A1 [2]	3.1a 1.1	B2 for 0.0416666...unsupported B0 for 0.042 unsupported	mark the final answer

Question			Answer	Marks	AOs		Guidance
8	(c)		$1 - \text{their } \frac{1}{24} \text{ evaluated}$	M1	3.1a		
			$1 - \left(\frac{23}{24}\right)^n > 0.95 \text{ FT}$	M1	2.1	allow use of $=$, $\geq$ or $\leq$	
			$\left(\frac{23}{24}\right)^n < 0.05 \text{ FT}$	M1	1.1	allow use of $=$ or $\leq$	
			$n = 71 \text{ cao}$	A1 [4]	2.2a	70.3890...unsupported but rounded to 1 or more dp implies M1M1M1	award full marks for 71 unsupported or from trial and improvement
9	(a)		$(2x + 3 - 1)^2$ or $(2x + 3)^2 - 2(2x + 3) + 1$ seen	M1	1.1	substitution	
			simplified to eg $4(x + 1)^2$ or $4x^2 + 8x + 4$ or $(2x + 2)^2$	A1	1.1	mark the final answer	ignore superfluous work on eg finding roots
			domain is $-1 < x < 0$	B1 [3]	1.1	from $2x + 3 > 1$	
9	(b)		$0 < \text{gf}(x) < 4$	B1 [1]	1.1		

Question			Answer	Marks	AOs		Guidance
9	(c)		factorise their $gf(x)$ to obtain perfect square or complete the square $y = 4(x + 1)^2$ or $(2x + 2)^2$ oe $(x + 1) = (\pm)\sqrt{\frac{y}{4}}$ oe $[(gf)^{-1}(x) =] \sqrt{\frac{x}{4}} - 1$ or $\frac{\sqrt{x}}{2} - 1$ oe domain is $0 < x < 4$	M1 A1 M1 A1 B1 [5]	3.1a 2.1 1.1 1.1 1.1	allow eg $2(x + 1)(2x + 2)$; may follow slip eg dividing by 4 FT FT their (b)	or $g^{-1}(x) = \sqrt{x} + 1$ or $f^{-1}(x) = \frac{1}{2}(x - 3)$ for M1 A1 for both correct M1 for their f^{-1}(their $\sqrt{x} + 1$) A1 for $(gf)^{-1}(x) = \sqrt{\frac{x}{4}} - 1$ or $\frac{\sqrt{x}}{2} - 1$ oe x and y may be interchanged for the first 3 marks but not for the final A1
10	(a)		[=]153	B1 [1]	1.1		
10	(b)		$z = \pm 1.645$ used their positive $1.645 = \frac{183 - \text{their } 153}{\sigma}$ oe (= 18.237...to 18.248...) $\sigma = 18.2$ cao	B1 M1 A1 [3]	1.1 2.1 1.1	or $\pm 1.644(85\dots)$	M0 if continuity correction used

Question			Answer	Marks	AOs		Guidance
10	(c)		$\left[\frac{16}{452} = \right] 0.035(398 \dots)$ their $P(X < 120)$ from $N(\text{their } 153, \text{their } 18.2^2)$ probability of 0.0349 to 0.0352 which agrees to 2 sf	B1 M1 A1 [3]	3.1b 1.1 2.2b	allow percentage M0 if continuity correction used allow percentage or B1 for their $P(X < 120)$ then M1 for $452 \times \text{their } 0.03490..$ then A1 for 15.77 to 15.91 which is close to 16 oe or B1 as main scheme then M1 for $\sigma = \frac{120 - 153}{\text{their } -1.809}$ and $18.3 \approx 18.2$ for A1	or B1 as main scheme then Invnorm(0.0353, their 153, their 18.2) for M1 NB 119.95 to 120.15 which is close to 120 oe for A1 or B1 as main scheme then $z = \frac{120 - \text{their } 153}{\text{their } 18.2}$ for M1 NB -1.809 to -1.813 and Invnorm(0.0353, 0, 1) to obtain -1.806 to -1.812 which is close to -1.809 to -1.813) for A1

Question			Answer	Marks	AOs		Guidance
10	(d)		$H_0: \mu = \text{their } 153$ $H_1: \mu < \text{their } 153$ μ is the population mean flight time from Magaluf to Liverpool use of $N(\text{their } 153, \frac{\text{their } 18.2^2}{24})$ to find $P(\bar{X} < 143.6)$ awrt 0.0057 to 0.0058... to 2 or more sf oe isw their 0.0057 correctly compared with 0.01 oe result is significant or reject H_0 or accept H_1 there is sufficient evidence to suggest at the 1% level that the mean flight time from Magaluf to Liverpool is less than 153 / mean flight time from Liverpool to Magaluf	B1 B1 M1* A1 M1dep* A1 A1 [7]	1.1 2.5 3.3 1.1 3.4 1.1 2.2b	for both hypotheses; may be stated in words, but need to see 153 for B1 and population mean for 2nd B1 or inv Norm(0.01, their 153, $\frac{\text{their } 18.2^2}{24}$) $\bar{X} < 144$ to 144.4 is critical region 143.6 correctly compared with their 144.36 FT their comparison FT their comparison	B0 if other parameter used unless clearly defined as population mean or $z = \frac{143.6 - \text{their } 153}{\frac{\text{their } 18.2}{\sqrt{24}}}$ $z = -2.5248..$ to $-2.5302...$ to 2 or more dp their z compared with -2.326 or -2.33 do not allow eg conclude / prove / indicate or other assertive statement instead of suggest

Question			Answer	Marks	AOs		Guidance
10	(e)		reduce the value of μ	B1	3.5c		eg take a bigger sample is insufficient
			increase the value of σ or σ^2	B1	3.5c	allow B1 for eg use new sample data to calculate new estimate for σ or σ^2	
				[2]			
11	(a)		differentiate to obtain $2x - 4$	B1	3.1a		allow one error
			$+ 1 \times \ln x + x \times \frac{1}{x}$ oe	M1	2.1	use of Product Rule	
			derivative = 0 oe seen and terms combined	A1	1.1	all correct	
			$2x - 3 + \ln x = 0$ www isw AG	A1	2.4		
				[4]			

Question			Answer	Marks	AOs		Guidance
11	(b)		any rearrangement to obtain $x = g(x)$ from given derivative = 0	M1*	2.1	allow sign error	eg $x = e^{2x-3}$
			$x = \frac{3 - \ln x}{2}$	A1	1.1	any correct rearrangement	need not see subscripts in iterative formula
			use of their $g(x_n) = \frac{3 - \ln x_n}{2}$ to obtain at least two iterates eg 2, 1.1534, 1.4286...	M1dep*	1.1	must see iterates	
			1.3500 cao	A1	2.2a		0 for 1.3500 unsupported
			 <i>Alternatively,</i>				trial and improvement does not score
			$x_{n+1} = x_n - \frac{2x_n - 3 + \ln x_n}{\text{their} \left(2 + \frac{1}{x_n}\right)}$	M1*	2.1	Newton-Rapshon iterative formula seen (not for solving $f(x) = 0$)	need not see subscripts in iterative formula
			use of their N-R formula to obtain $x_1, x_2, \dots$ eg 1.5, 1.34795, 1.34996, ..., 1.349962	A1 M1dep*	1.1 1.1	formula all correct must see iterates	
			1.3500 cao	A1 [4]	2.2a		

Question			Answer	Marks	AOs		Guidance
12	(a)		$0.94^n < k$ or $0.06^n < k$ seen $k = 0.025$ used in inequality as above 60	M1 B1 A1 [3]	3.4 1.1 2.2a	NB 59.617.. or 1.311...to 1 or more dp if unsupported implies M1B1	allow = instead of < 60 unsupported or from trial and improvement scores 3

Question			Answer	Marks	AOs		Guidance
12	(b)		$H_0 : p = 0.06$ allow equivalent in words $H_1 : p \neq 0.06$ p is the probability that a jaguar chosen at random is a black panther / has black coat use of B(83, 0.06) to obtain $P(X \leq K)$ oe cdfBinomial(83, 0.06, 9) = 0.973 to 0.97321... or $1 - \text{cdfBinomial}(83, 0.06, 9) = 0.02679$ to 0.027 $1 -$ their $P(X \leq K)$ compared with 0.025 or their $P(X \leq K)$ compared with 0.975 oe result is not significant or do not reject H_0 or reject H_1 there is insufficient evidence at the 5% level to suggest that the probability that a jaguar selected at random from this population is a black panther is not 0.06	B1 B1 M1* A1 M1dep* A1 A1 [7]	1.1 2.5 3.3 1.1 3.4 1.1 2.2b	 or p is the proportion of jaguars that are black panthers / have a black coat not $P(X = K)$ or critical region is $X \geq 11$ (ignore lower tail) eg 10 compared with their critical region oe must have the correct probability or correct critical region for the last two A marks	 NB $P(X \leq 10) = .98927...$ for comparison of their $P(X > K)$ with 0.025 or their $P(X \leq K)$ with 0.975 or stating whether 10 is in their critical region allow accept H_0 do not allow eg conclude / prove / indicate or other assertive statement instead of suggest
13	(a)		3	B1 [1]	3.3		

Question			Answer	Marks	AOs		Guidance
13	(b)		$[8 \times 3^4 =] 648$	B1 [1]	1.1		
13	(c)		$\frac{8(3^n - 1)}{3 - 1}$ $= 4(3^n - 1)$ or $-4(1 - 3^n)$	B1 [1]	3.4	use of formula for sum of gp mark the final answer	or $4 \times 3^n - 4$
13	(d)		their $4(3^n - 1) = 185\,207$ or $3^n = 46303(.75)$ awrt 9.8 cao [=] 9	M1 A1 A1 [3]	3.1a 1.1 3.2a	M0 for eg $8 \times 3^{n-1}$ no FT available here not from wrong working	allow use of $<$ or $\leq$ for up to 3 marks allow M1 only for use of $>$ or $\geq$ or $3^9 = 19683$ and $3^{10} = 59049$ seen for M1 then A1 (comparison with 46 303)
13	(e)		unlikely because eg some of the population will be immune to the virus eg some of the population will not be exposed to the virus eg medical intervention eg extrapolation eg movement of people in and out of town	B1 [1]	3.5b	any sensible reason	it's unlikely that everyone will be affected oe is insufficient

Question			Answer	Marks	AOs		Guidance
14	(a)		the data was not available for all countries oe	B1 [1]	2.4		
14	(b)		use of $Q_1 - 1.5 \times (Q_3 - Q_1)$ and $Q_3 + 1.5 \times (Q_3 - Q_1)$ seen for either set $4.135 < 6.28$ and $15.775 > 14.46$ $0.38 < 3.58$ and $18.86 > 14.89$	M1 A1 A1 [3]	3.1b 1.1 1.1	if A0A0 allow SC1 for 4.135, 15.775, 0.38 and 18.86 all seen	
14	(c)		22 954 isw	B1 [1]	3.1b	allow 22 955, 22 950 or 23 000	NB $6411776 \times \frac{3.58}{1000}$
14	(d)		there are almost certainly more “old” people in the population oe	B1 [1]	2.4		

Question			Answer	Marks	AOs		Guidance
14	(e)		in African countries there is a negative association / relationship between (or negative correlation between the ranks of) median age and crude death rate, but in Europe there seems to be a positive association / relationship between (or positive correlation between the ranks of) median age and crude death rate	B1	2.4	do not allow “negative correlation” and / or “positive correlation”	comment comparing and contrasting type of relationship in both continents for B1 , and one comment comparing and contrasting strength of relationship in both continents for B1
			the “association” / “relationship between” or “correlation between the ranks of” median age and crude death rate (appears to be) stronger in Europe	B1 [2]	2.4	allow B1 both relationships are weak oe	allow equivalent explanations in words eg as median age increases crude death rates decrease in Africa and similar for Europe

Question			Answer	Marks	AOs		Guidance
15			$\mu = 35$ soi	B1	1.2	by symmetry	may be embedded in N(35,4 ²) or N(35,16) M0 if continuity corrections used their 0.7887 must be from use of N(35, 4 ²)
			$\sigma = 4$ seen	B1	3.1b	$35 - 31 = 4$	
			use of N(35, 4 ²) to obtain a value for P(30 < X < 40)	M1	2.1		
			[cdfNormal(30, 40, 35, 4) =] 0.788700... BC to 2 or more dp	A1	1.1		
			use of Y ~B(50, their 0.7887) to find P(Y ≤ K) or P(Y ≥ K)	M1	3.1b	eg cdfBinomial(50,0.7887,45,50)	
			[1 – P(Y ≤ 44)] = 0.032 to 0.034 BC	A1	1.1		
				[6]			

OCR (Oxford Cambridge and RSA Examinations)
The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

OCR Customer Contact Centre

Education and Learning

Telephone: 01223 553998

Facsimile: 01223 552627

Email: general.qualifications@ocr.org.uk

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