



Oxford Cambridge and RSA

Monday 05 October 2020 – Afternoon

AS Level Further Mathematics B (MEI)

Y410/01 Core Pure

Time allowed: 1 hour 15 minutes



You must have:

- the Printed Answer Booklet
- the Formulae Booklet for Further Mathematics B (MEI)
- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

- The total mark for this paper is **60**.
- The marks for each question are shown in brackets [].
- This document has **4** pages.

ADVICE

- Read each question carefully before you start your answer.

Answer **all** the questions.

1 In this question you must show detailed reasoning.

Find $\sum_{r=2}^{50} \left(\frac{1}{r-1} - \frac{1}{r+1} \right)$, expressing the answer as an exact fraction. [3]

2 Fig. 2 shows two complex numbers z_1 and z_2 represented on an Argand diagram.

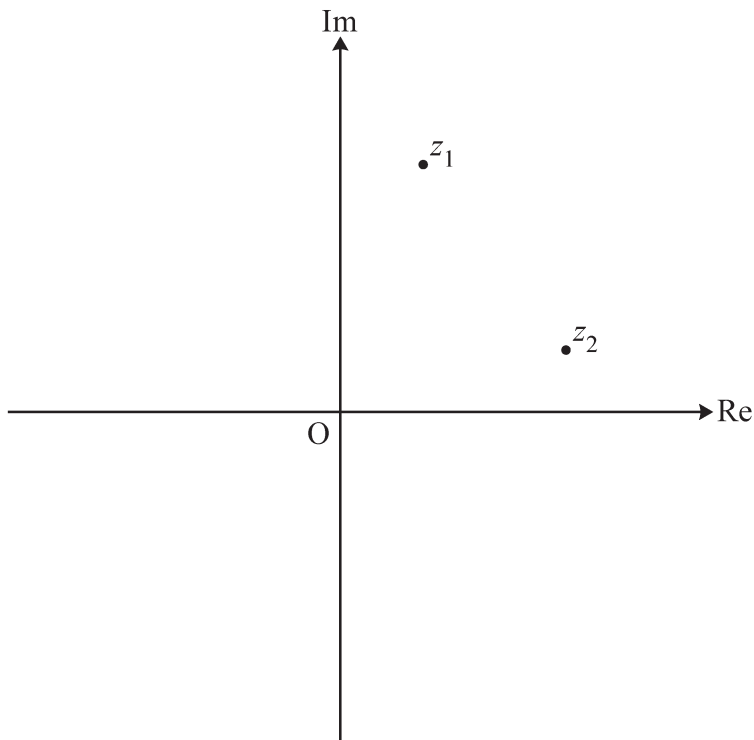


Fig. 2

(a) On the copy of Fig. 2 in the Printed Answer Booklet, mark points representing each of the following complex numbers.

- z_1^*
- $z_2 - z_1$

[2]

(b) **In this question you must show detailed reasoning.**

In the case where $z_1 = 1 + 2i$ and $z_2 = 3 + i$, find $\frac{z_2 - z_1}{z_1^*}$ in the form $a + ib$, where a and b are real numbers. [2]

3 In this question you must show detailed reasoning.

The roots of the equation $x^2 - 2x + 4 = 0$ are α and β .

(a) Find α and β in modulus-argument form. [4]

(b) Hence or otherwise show that α and β are both roots of $x^3 + \lambda = 0$, where λ is a real constant to be determined. [3]

4 The matrix $\mathbf{M}$ is $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

(a) (i) Calculate $\det \mathbf{M}$. [1]

(ii) State two geometrical consequences of this value for the transformation associated with $\mathbf{M}$. [2]

(b) Describe fully the transformation associated with $\mathbf{M}$. [1]

5 You are given that $u_1 = 5$ and $u_{n+1} = u_n + 2n + 4$.

Prove by induction that $u_n = n^2 + 3n + 1$ for all positive integers n . [6]

6 The matrices $\mathbf{M}$ and $\mathbf{N}$ are $\begin{pmatrix} \lambda & 2 \\ 2 & \lambda \end{pmatrix}$ and $\begin{pmatrix} \mu & 1 \\ 1 & \mu \end{pmatrix}$ respectively, where λ and μ are constants.

(a) Investigate whether $\mathbf{M}$ and $\mathbf{N}$ are commutative under multiplication. [2]

(b) You are now given that $\mathbf{MN} = \mathbf{I}$.

(i) Write down a relationship between $\det \mathbf{M}$ and $\det \mathbf{N}$. [1]

(ii) Given that $\lambda > 0$, find the exact values of λ and μ . [3]

(iii) Hence verify your answer to part (i). [2]

7 In the quartic equation $2x^4 - 20x^3 + ax^2 + bx + 250 = 0$, the coefficients a and b are real. One root of the equation is $2 + i$.

Find the other roots. [7]

8 (a) The matrix $\mathbf{M}$ is $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$.

(i) Find $\mathbf{M}^2$. [1]

(ii) Write down the transformation represented by $\mathbf{M}$. [1]

(iii) Hence state the geometrical significance of the result of part (i). [1]

(b) The matrix $\mathbf{N}$ is $\begin{pmatrix} k+1 & 0 \\ k & k+2 \end{pmatrix}$, where k is a constant.

Using determinants, investigate whether $\mathbf{N}$ can represent a reflection. [4]

9 Three planes have equations

$$kx + y - 2z = 0$$

$$2x + 3y - 6z = -5$$

$$3x - 2y + 5z = 1$$

where k is a constant.

Investigate the arrangement of the planes for each of the following cases. If in either case the planes meet at a unique point, find the coordinates of that point.

(a) $k = -1$ [3]

(b) $k = \frac{2}{3}$ [4]

10 A vector $\mathbf{v}$ has magnitude 1 unit. The angle between $\mathbf{v}$ and the positive z -axis is 60° , and $\mathbf{v}$ is parallel to the plane $x - 2y = 0$.

Given that $\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, where a , b and c are all positive, find $\mathbf{v}$. [7]

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

Copyright Information

OCR is committed to seeking permission to reproduce all third-party content that it uses in its assessment materials. OCR has attempted to identify and contact all copyright holders whose work is used in this paper. To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced in the OCR Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download from our public website (www.ocr.org.uk) after the live examination series.

If OCR has unwittingly failed to correctly acknowledge or clear any third-party content in this assessment material, OCR will be happy to correct its mistake at the earliest possible opportunity.

For queries or further information please contact The OCR Copyright Team, The Triangle Building, Shaftesbury Road, Cambridge CB2 8EA.

OCR is part of the Cambridge Assessment Group; Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.