



Practice Paper – Set 1

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 100

**Version - Final
Last updated 05/12/17**

This document consists of 16 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction In this question you must show detailed reasoning appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B

Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is

worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as *cao* may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.
- k Anything in the mark scheme which is in square brackets [...] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like.

Question			Answer	Marks	AOs	Guidance	
1			$\frac{dy}{dx} = 4(3x^2 + 5)^3 \times 6x$	M1	1.1a	Use of chain rule attempted	
				A1	1.1b	6x soi	
			$\frac{dy}{dx} = 24x(3x^2 + 5)^3$	A1	1.1b		
				[3]			
2			DR				
			$\int_0^{\frac{1}{3}\pi} (3\sin 3x - \sin 3x) dx$	M1	1.1a	Integrating the difference of the functions	
			$\left[-\frac{2}{3}\cos 3x\right]_0^{\frac{1}{3}\pi}$	B1	1.1b	$\int \sin 3x dx = -\frac{1}{3}\cos 3x$ oe soi	
			$= -\frac{2}{3}(\cos \pi - \cos 0)$	B1	3.1a	Limits used or indicated on a diagram	
			$= -\frac{2}{3}(-1 - 1) = \frac{4}{3}$ AG	A1	2.1	Must be clearly shown	
				[4]			
3			Sequence of the form $\ln a, \ln ar, \ln ar^2 \dots$	M1	2.5	Use of notation for geometric sequence	
			$\ln a, \ln a + \ln r, \ln a + 2\ln r \dots$	M1	3.1a	Laws of indices used	
			Arithmetic sequence with common difference $\ln r$	E1	2.2a	Clear indication of common difference	
				[3]			

Question			Answer	Marks	AOs	Guidance	
4			$v = \frac{ds}{dt} = 3 + 0.3t^2$ and $a = \frac{dv}{dt} = 0.3 \times 2t$	M1	1.1a	Attempt to differentiate twice	
			$a = 0.6t$	A1	2.1		
			Acceleration is a function of t so is not constant	E1	2.2a	Must be clearly argued	
			Alternative method				
			For constant acceleration, $s = ut + \frac{1}{2}at^2$ (+c)	M1		Comparing with standard formula	
			i.e. s is a quadratic function of t	A1		Explicit identification of quadratic, oe	
			So this cubic function is not constant acceleration	E1		Deduction clearly stated	
				[3]			
5			Distance of 50 g piece from pivot is $(15 - x)$ cm	B1	3.3	soi; may be on the diagram	
			Moments about pivot: $20 \times 15 = 50 \times (15 - x)$ oe	M1	3.1b	Allow any consistent units of weight (or mass) and length	Condone e.g. 20g with no units for g stated
				A1	2.1	Correct (unsimplified) equation	
			$x = 9$	A1	1.1b		
				[4]			
6	(i)		Area of triangle $= \frac{1}{2} \times 10^2 \sin \frac{1}{6} \pi$ (= 25)	B1	1.1a	Correct unsimplified expression soi	
			$AC = 2 \times 10 \sin \frac{1}{12} \pi$ (= 5.17638)	M1	3.1a	Or use of cosine rule	
			Area of semicircle $= \frac{1}{2} \pi \times \left(\frac{5.17638}{2} \right)^2$ (= 10.5223)	M1	1.1b	Use of $\frac{1}{2} \pi \times (\text{half of their } AC)^2$	
			Total area = 35.5 cm ²	A1	1.1a		
				[4]			
	(ii)		$50\theta = 35.52$	M1	1.1a	Equating $\frac{1}{2} r^2 \theta$ to their answer from (i)	
			$\theta = 0.710$ to 3sf	A1	1.1b		
				[2]			

Question		Answer	Marks	AOs	Guidance	
7		T_1 and T_2 are tensions in wire and string respectively; the globe has mass m Resolve vertically: $T_1 \cos \theta = mg$ $T_1 = \frac{mg}{\cos \theta}$ Resolve horizontally: $T_2 = T_1 \sin \theta$ $T_2 = \frac{mg}{\cos \theta} \times \sin \theta = mg \tan \theta$ $mg \tan \theta < mg$ for $\tan \theta < 1$ $\theta < 45^\circ$ Alternative method Triangle of forces Maximum T_2 is when $T_2 = W$ In this case $\theta = \arctan(1)$ Hence $\theta < 45^\circ$	M1 M1 M1 A1 M1 M1 M1 A1 [4]	3.1b 3.3 2.1 2.2a	Allow sin/cos interchange for 1st M1 only Use of trig identity Inequality clearly stated Forces shown, correct arrow directions Equating T_2 and weight for limiting case Use of trigonometry or isosceles triangle Inequality clearly stated	

Question			Answer	Marks	AOs	Guidance	
8	(i)		$(1-2x)^{-\frac{1}{2}} \approx 1 + \left(-\frac{1}{2}\right)(-2x) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-2x)^2 + \dots$ $= 1 + x + \frac{3}{2}x^2 \quad \text{AG}$	M1	1.1b	Correct form required; allow sign errors	
				A1 [2]	2.1	Must be correctly obtained	
	(ii)		Valid when $ x < \frac{1}{2}$	E1 [1]	2.3		
	(iii)		$(1+2x)^{\frac{1}{2}} = 1 + x - \frac{1}{2}x^2 + \dots$ $\left(1 + x - \frac{1}{2}x^2\right)\left(1 + x + \frac{3}{2}x^2\right)$ $= 1 + 2x + 2x^2$	B1	1.1a	Product of their expansions attempted	
				M1	1.1a		
				A1	1.1b		
			Alternative method $\frac{\sqrt{1+2x}}{\sqrt{1-2x}} = \frac{\sqrt{(1+2x)(1+2x)}}{\sqrt{(1-2x)(1+2x)}} = \frac{1+2x}{\sqrt{1-4x^2}}$ $= (1+2x)\left(1 + \left(-\frac{1}{2}\right)(-4x^2) + \dots\right)$ $= 1 + 2x + 2x^2$	M1		Converting to rational numerator form	
				M1		Expand denominator and multiply out	
				A1 [3]			
	(iv)		$\sqrt{\frac{1.1}{0.9}} = \frac{\sqrt{11}}{3} \approx 1 + 2 \times \frac{1}{20} + 2 \times \left(\frac{1}{20}\right)^2$ $\sqrt{11} \approx 3.315$	M1	2.1	Obtaining an expression involving $\sqrt{11}$	
				A1 [2]	2.2a	Or $\frac{663}{200}$	

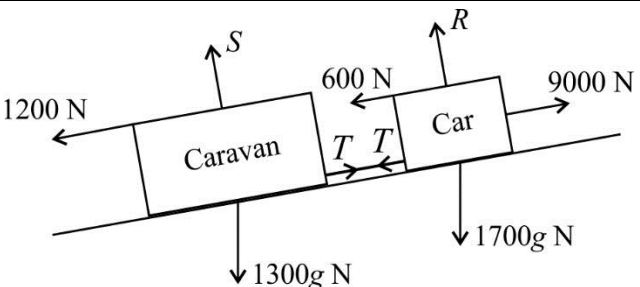
Question			Answer	Marks	AOs	Guidance		
9			Horizontal motion: $x = (8\cos 35)t$	B1	3.3			
			Vertical motion: $y = (8\sin 35)t - 4.9t^2 + 1$	M1	3.3			Allow for RHS with first two terms only
			Time to wall: $(8\cos 35)t = 4$	M1	3.1b			Attempt to find t when $x = 4$
			$t = 0.6104$	A1	1.1b			Allow 0.61 or better
			Height at this time: $(8\sin 35)0.6104 - 4.9(0.6104)^2 + 1$	M1	1.1a			Substitute their value of t in their y
			$y = 1.975$	A1	1.1b			Allow 0.975 only if compared with 0.9
			This is between 1.9 and 2.0 so the stone hits the can	E1	3.2a			Comment must be supported by evidence
			Alternative method					
			Horizontal motion: $x = (8\cos 35)t$	B1		May be implied if trajectory eqn is quoted		
			Vertical motion: $y = (8\sin 35)t - 4.9t^2 + 1$	M1		May be implied if trajectory eqn is quoted		
			Trajectory: $y = (8\sin 35)\left(\frac{x}{8\cos 35}\right) - 4.9\left(\frac{x}{8\cos 35}\right)^2 + 1$	M1				
				A1		Allow without the '+1' if subsequent work is all consistent with origin at height 1		
			Height at $x = 4$: $(8\sin 35)\left(\frac{4}{8\cos 35}\right) - 4.9\left(\frac{4}{8\cos 35}\right)^2 + 1$	M1				
			$= 1.975$	A1				
			This is between 1.9 and 2.0 so the stone hits the can	E1		Comment must be supported by evidence		
	[7]							

Question			Answer	Marks	AOs	Guidance	
10	(i)		$\sec \theta - \cos \theta \equiv \frac{1}{\cos \theta} - \cos \theta$	B1	1.2	Use of definition of sec	
			$\equiv \frac{1 - \cos^2 \theta}{\cos \theta}$	M1	2.1	Manipulation of fractions	
			$\equiv \frac{\sin^2 \theta}{\cos \theta} \equiv \frac{\sin \theta}{\cos \theta} \sin \theta$	M1	1.2	Use of at least one trig identity	
			$\equiv \tan \theta \sin \theta$ AG	E1	2.1	Clear and complete argument	
				[4]			
	(ii)		$\tan \theta \sin \theta = \frac{1}{2} \tan \theta \Rightarrow \tan \theta \left(\sin \theta - \frac{1}{2} \right) = 0$	M1	1.1a	Factorising	SC1 for sole answer $\frac{1}{6} \pi$ www
			$\tan \theta = 0$ gives $\theta = 0, \pi$	A1	1.1b	Both values	
			$\sin \theta = \frac{1}{2}$ gives $\theta = \frac{1}{6} \pi, \frac{5}{6} \pi$	M1	1.1a	At least one value from arcsin	
				A1	1.1b	Both values correct	
			Alternative method				
			$1 - \cos^2 \theta = \frac{1}{2} \sin \theta \Rightarrow \sin^2 \theta = \frac{1}{2} \sin \theta$	M1		Obtaining quadratic in $\sin \theta$	
			$\sin \theta = 0$ gives $\theta = 0, \pi$	A1		Both values	
			$\sin \theta = \frac{1}{2}$ gives $\theta = \frac{1}{6} \pi, \frac{5}{6} \pi$	M1		At least one value from arcsin	
				A1		Both values correct	
				[4]			

Question			Answer	Marks	AOs	Guidance	
11	(i)		$21 = \frac{1}{2}(u+0)14$ $u = 3$ AG	M1 A1 [2]	1.1a 2.1	Use of <i>suvat</i> equation(s) to find u	
	(ii)		$0 = 3^2 + 2a \times 21$ $a = -\frac{3}{14}$ Magnitude of force = $ ma = 18 \times \frac{3}{14}$ $= \frac{27}{7} = 3.86$ N	M1 M1 A1 [3]	3.1b 1.1a 1.1b	Use of <i>suvat</i> to find acceleration Use of Newton II Must be magnitude (positive)	
	(iii)		$N = 18g$ $3.86 = \mu \times 18g$ $\mu = 0.022$ to 2 sf	B1 M1 A1 [3]	3.1b 3.4 1.1b	soi Use of $F = \mu N$ to find μ Must be given correct to 2sf	

Question			Answer	Marks	AOs	Guidance	
12	(i)		When $t = 3$, $ \mathbf{v} = 1.5\mathbf{i} + 1.5\mathbf{j} = \sqrt{1.5^2 + 1.5^2}$ Speed is $\frac{3}{2}\sqrt{2} = 2.12 \text{ m s}^{-1}$	M1 A1 [2]	1.1b 1.1b	Substitute for t and attempt modulus	
	(ii)		$\mathbf{r} = \int \mathbf{v} \, dt = 1.5t\mathbf{i} + 0.5 \times \frac{t^2}{2} \mathbf{j} + \mathbf{c} = 1.5t\mathbf{i} + 0.25t^2\mathbf{j} + \mathbf{c}$ $t = 0$ and $\mathbf{r} = 2\mathbf{j} \Rightarrow \mathbf{c} = 2\mathbf{j}$ $\mathbf{r} = 1.5t\mathbf{i} + (0.25t^2 + 2)\mathbf{j}$	M1 A1 A1 [3]	1.1a 1.1b 2.1	Attempt to integrate both terms Allow without $+ \mathbf{c}$ Terms need not be collected	
	(iii)		$x = 1.5t$ and $y = 0.25t^2 + 2$ $y = 0.25\left(\frac{x}{1.5}\right)^2 + 2$ $y = \frac{x^2}{9} + 2$ AG	M1 M1 A1 [3]	2.1 2.1 1.1b	Allow for either equation Eliminate t Must be fully justified	

Question			Answer	Marks	AOs	Guidance	
13	(i)		$a = 40$ $c = 180$ $30 = 40 - b(135 - 180)^2 \Rightarrow b = \frac{2}{405}$ AG	B1 B1 B1 [3]	3.3 3.3 2.1	At least one intermediate step needed	
	(ii)	(A)	15.8	B1 [1]	3.4	Allow integer answers 15 or 16	
		(B)	250 is outside the interval which provides the data and so may be extrapolating too far	E1 [1]	3.5a		
	(iii)	(A)	$t < 90 \Rightarrow N < 0$ which is not meaningful	E1 [1]	2.4		
		(B)	$t > 270$	B1 [1]	3.5b		
	(iv)		Reflection in t -axis or translation by $\begin{pmatrix} 180 \\ 0 \end{pmatrix}$ or $\begin{pmatrix} -180 \\ 0 \end{pmatrix}$ Stretch in the y -direction, scale factor 40	B1 B1 [2]	1.1a 1.1a	Vector notation required for translation	Accept stretch in the y -direction, scale factor -40 , for B2
	(v)		$N = -40\cos t$ or $N = 40\cos(t - 180)$	B1 [1]	3.3	oe, e.g. $N = 40\cos(t + 180)$	
	(vi)	(A)	Stretch graph in t -direction by scale factor $\frac{365}{360}$ or modify equation to $N = -40\cos\left(\frac{360}{365}t\right)$	B1 [1]	3.5c	oe, e.g. $N = 40\cos\left(\frac{360}{365}t - 180\right)$; FT from their (v) for modified equations	
		(B)	Maximum point moves to (182.5, 40) or as consistent with their answer to (A)	B1 [1]	3.4	N.B: $N = 40\cos\left(\frac{360}{365}(t - 180)\right)$ oe will leave maximum point unchanged	

Question			Answer	Marks	AOs	Guidance	
14	(i)		 Weights and normal reactions for each correctly shown 9000 N and force T in coupling correctly shown Resistances 600 N and 1200 N correctly shown	B1 B1 B1 [3]	1.1a 3.3 3.3	The two force diagrams may be shown separately	
	(ii)		Resolve weights parallel to the slope $9000 - 600 - 1200 - 3000g \sin 8 = 3000a$ $7200 - 4091.689 = 3000a \Rightarrow a = 1.04 \text{ m s}^{-2}$	M1 M1 A1 A1 [4]	3.1a 3.3 1.1b 1.1b	Allow sin/cos errors here Newton II; all non-weight terms required All terms correct cao	
	(iii)		$T - 1200 - 1300g \sin 8 = 1300 \times 1.04$ or $9000 - 1700g \sin 8 - 600 - T = 1700 \times 1.04$ $T = 4320 \text{ N}$ (answer using a unrounded)	M1 A1 A1FT [3]	3.4 1.1a 1.1b	Resolving parallel to the slope for car or caravan; all forces present, correct mass All terms in equation correct (for their a) FT their rounded/unrounded a	

Question			Answer	Marks	AOs	Guidance	
15	(i)		$\int \frac{1}{y(1+y)} dy = \int (1-x) dx$	M1	1.1a	Separating the variables	
			$\frac{1}{y(1+y)} = \frac{1}{y} - \frac{1}{1+y}$	M1	3.1a	Correct form $\frac{A}{y} + \frac{B}{1+y}$ used	
			$\ln y - \ln(1+y) = x - \frac{1}{2}x^2 + c$	A1	1.1b	Correct partial fractions	
			$\ln 1 - \ln(1+1) = 1 - \frac{1}{2} + c$	A1	1.1b	oe, e.g. RHS $-\frac{1}{2}(1-x)^2 + c$	
			$c = -\frac{1}{2} - \ln 2$	M1	1.1a	Use of (1, 1) to find c	
			$\ln y - \ln(1+y) = x - \frac{1}{2}x^2 - \frac{1}{2} - \ln 2$	A1	1.1b		
			$\ln\left(\frac{2y}{1+y}\right) = x - \frac{1}{2}x^2 - \frac{1}{2}$				
			$\frac{2y}{1+y} = e^{x - \frac{1}{2}x^2 - \frac{1}{2}}$	M1	1.1a	Writing in non-logarithmic form	
			$2y = (1+y)e^{x - \frac{1}{2}x^2 - \frac{1}{2}}$				
			$y = \frac{e^{x - \frac{1}{2}x^2 - \frac{1}{2}}}{2 - e^{x - \frac{1}{2}x^2 - \frac{1}{2}}}$	M1	1.1a	Making y the subject	
				A1	1.1b	Correct $y = f(x)$	
				[9]			

Question			Answer	Marks	AOs	Guidance	
	(ii)		At (1, 1), $\frac{dy}{dx} = 1 \times 2(1-1) = 0$	B1	3.1a		
			Near (1, 1), $y(1+y)$ is positive and $(1-x)$ is positive for $x < 1$ but negative for $x > 1$	M1	1.1b	Determining sign of gradient on each side	
			$\frac{dy}{dx} > 0$ for $x < 1$ and $\frac{dy}{dx} < 0$ for $x > 1$ so maximum point	E1	2.2a	Clear deduction seen	
			Alternative method				
			At (1, 1), $\frac{dy}{dx} = 1 \times 2(1-1) = 0$	B1			
			$\frac{d^2y}{dx^2} = (1+2y)\frac{dy}{dx}(1-x) + (y+y^2)(-1)$	M1		Differentiation using product rule and evaluation at (1, 1)	
			$\frac{d^2y}{dx^2} = 3 \times 0 + 2(-1) = -2 < 0$			Condone -2 not seen if clearly negative	
			(1, 1) is a maximum point	E1		Must follow both 1st and 2nd derivatives	
				[3]			