Oxford Cambridge and RSA

Practice Paper – Set 1

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

MARK SCHEME

Duration: 2 hours

MAXIMUM MARK 75

Version: Final

Last updated: 06/12/2017

This document consists of 13 pages

Text Instructions

1. Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

2. Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasion, but shows what a complete solution might look like.

l. Crossed Out Responses
Where a candidate has crossed out a response and provided a clear alternative then the crossed out response is not marked. Where no alternative response has been provided, examiners may give candidates the benefit of the doubt and mark the crossed out response where legible.

Contradictory Responses
When a candidate provides contradictory responses, then no mark should be awarded, even if one of the answers is correct.

m. Award No Response (NR) if:
- there is nothing written in the answer space.

Award Zero '0' if:
- anything is written in the answer space and is not worthy of credit (this includes text and symbols).

Team Leaders must confirm the correct use of the NR button with their markers before live marking commences and should check this when reviewing scripts.

Question			Answer	Marks	AOs	Guidance	
1	(i)		[g(x) =] $\lvert x \rvert$	B1 [1]	1.1		
	(ii)		All non-negative real numbers	B1 [1]	1.1	Allow any reasonable notation, e.g. $y \geq 0$	
	(iii)		**Either**				
			$\lvert 3x - 1 \rvert > 1$				
			$\Rightarrow 3x - 1 > 1$	M1	1.1		
			or $3x - 1 < -1$	M1	1.1		
			Or				
			$(3x - 1)^2 > 1$	M1		Allow for sketch of $y = (3x - 1)^2$ and $y = 1$	
			$9x^2 - 6x > 0 \Rightarrow 3x(3x - 2) > 0$	M1		Allow for critical values $x = 0, \frac{2}{3}$	
			Hence $x > \frac{2}{3}, x < 0$	A1	1.1	For either of these inequalities	
			$\{x : x < 0\} \cup \left\{x : x > \frac{2}{3}\right\}$	A1 [4]	2.5	Final answer must be in this set notation form or else stated as '$x < 0$ or $x > \frac{2}{3}$'	

2			**DR**				
			Substitute $x = 0$ or $y = 0$ in $2x + y = 6$	M1	3.1a		
			Line crosses axes at (0, 6) and (3, 0)	A1	1.1		
			Quadratic with factor $(x - 3)$	M1	3.1a		
			Repeated factor $(x - 3)$	M1	1.1		
			$y = \frac{2}{3}(x - 3)^2$ oe	A1 [5]	2.1	Must follow from clear reasoning	

Question			Answer	Marks	AOs	Guidance	
3			OP = OQ = 6	**M1**	**3.1a**	Use area of triangle to find OP and OQ	
			Centre of circle is at (3, 3)	**M1**	**3.1a**	Midpoint of PQ	
			$r^2 = 3^2 + 3^2$	**M1**	**1.1**		
			$= 18$	**A1**	**1.1**		
			$(x-3)^2 + (y-3)^2 = 18$	**A1**	**1.1**	oe isw	
				[5]			

4			Gradient of chord $= \dfrac{(x+h)^3 - x^3}{h}$ oe	**M1**	**2.1**		
			$= \dfrac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$	**M1**	**1.1a**	Expansion of cubic attempted, with at least some terms correct	Not just $x^3 + h^3$
				A1	**1.1**	Completely correct expansion	
			Derivative is limit of gradient of chord as $h \to 0$	**E1**	**2.4**		
			$3x^2 + 3xh + h^2 \to 3x^2$ **AG**	**B1**	**2.2a**		
				[5]			

Question			Answer	Marks	AOs	Guidance	
5	(i)		**DR**				
			At intersections, $x^2 - kx = 3(k + 1) + kx - x^2$	**M1**	**3.1a**	Forming quadratic	
			$2x^2 - 2kx - 3(k + 1) = 0$	**M1**	**1.1**	0 on one side of quadratic	
			For touching, '$b^2 - 4ac = 0$'	**M1**	**2.1**	Forming quadratic from discriminant	
			$4k^2 + 24(k + 1) = 0 \Rightarrow k^2 + 6k + 6 = 0$	**A1**	**2.2a**	Correct quadratic in 3-term form	
			$k = \frac{-6 \pm \sqrt{12}}{2}$	**B1**	**1.1**	Use of formula or completing the square	$(k+3)^2 = 3$
			$k = -3 \pm \sqrt{3}$	**B1**	**1.1**		
				[6]			
	(ii)		On y-axis $x = 0$, so $y = 0^2 - 0k = 0$	**M1**	**3.1a**	Use of $x = 0$ to find y	
			If they cross on the y-axis, they cross at the origin	**B1**	**2.2a**		
			$0 = 3(k + 1) + k0 - 0^2$	**M1**	**1.1**	Use of $x = 0$ and $y = 0$	
			$k = -1$ [so a value exists]	**A1**	**2.1**		
				[4]			

6			$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$	**M1**	**1.1**	Attempt to subtract two position vectors to get the vector for a side	Allow alternative vector notation
			$= 5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$	**A1**	**1.1**		
			$\overrightarrow{DC} = 5\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}$ oe	**M1**	**3.1a**	Equating opposite side to vector found **or** finding $\overrightarrow{AC}$ and using $\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$ oe	Allow $\overrightarrow{CD}$ here, but no ft from $\overrightarrow{CD}$ for final answer
			$\overrightarrow{OD} = 5\mathbf{j} + 7\mathbf{k}$	**A1**	**2.2a**		
				[4]			

Question			Answer	Marks	AOs	Guidance	
7	(i)		**DR**				
			$\frac{dy}{dx} = 4x^3 - 6x$	**M1**	**1.1**	Attempt to differentiate with at least one term correct	
			When $x = 1$, $\frac{dy}{dx} = -2$	**A1**	**1.1**		
			Gradient of normal is $\frac{1}{2}$	**M1**	**1.1**	Use of $m_1 m_2 = -1$	
			Equation is $y = \frac{1}{2}(x-1)$	**A1**	**1.1**	oe	
				[4]			
	(ii)		**DR**				
			Curve is part **(i)** translated up	**M1**	**3.1a**		
			by $\frac{1}{2}$	**M1**	**1.1**		
			$c = 2.5$	**A1**	**2.2a**		
			Alternative method				
			Equation of normal is $y = \frac{1}{2}x$	**M1**			
			Point on curve is $\left(1, \frac{1}{2}\right)$	**M1**			
			$c = 2.5$	**A1**			
				[3]			

Question			Answer	Marks	AOs	Guidance	
8			**DR**				
			$\dfrac{b^2-13}{1-\dfrac{1}{b}}=-6$	**M1**	**3.1a**	Use of sum to infinity to form an equation	
			$\dfrac{b(b^2-13)}{b-1}=-6$	**M1**	**3.1a**	Starting to clear fractions	
			$b(b^2-13)=-6(b-1)$	**A1**	**1.1**	Correct equation without fractions	
			$b^3-7b-6=0$	**M1**	**1.1**	Cubic with zero on one side	
			$(-1)^3-7\times(-1)-6=0$	**M1**	**2.1**	Use of factor theorem to search for factor	
			$(b+1)(b^2-b-6)=0$	**A1**	**1.1**	Correct factors	
			$(b+1)(b+2)(b-3)=0$	**M1**	**1.1**	Method for solving quadratic	
			Roots –1, –2, 3				
			–1 cannot be the common ratio of a geometric sequence with a sum to infinity	**B1**	**2.3**	Rejection of root that does not make sense in the context	
			Possible common ratios are $-\frac{1}{2}$ and $\frac{1}{3}$	**A1FT**	**3.2a**	Follow through their values of *b*	
				[9]			

Question			Answer	Marks	AOs	Guidance	
9	(i)		$2\cos\theta + 3\sin\theta \equiv R\sin(\theta+\alpha) \Rightarrow R\cos\alpha = 3, R\sin\alpha = 2$	M1	1.1a		
			So $R^2 = 13 \Rightarrow R = \sqrt{13}$	B1	1.1		
			and $\tan\alpha = \frac{2}{3}$	M1	1.1		
			$\Rightarrow \alpha = 0.588$	A1	1.1		
				[4]			
	(ii)		$k + 2\cos x + 3\sin x > 0$ [for all x]	B1	3.1a	oe	
			$\sqrt{13}\sin(x+0.588) + k > 0$	M1	1.1	Use of expression from part **(i)**	
			Minimum value of LHS is $k - \sqrt{13}$	M1	3.1a	Attempt to find minimum value	May be by calculus
			$k > \sqrt{13}$	A1	2.2a		
				[4]			
	(iii)		$k + 2\cos x + 3\sin x > 0 \Rightarrow \dfrac{1}{k + 2\cos x + 3\sin x} > 0$	E1	2.4	oe; accept e.g. statement that the reciprocal of a positive number is positive	
				[1]			

Question			Answer	Marks	AOs	Guidance	
10			$\int_1^8 \ln x \, dx = [x\ln x]_1^8 - \int_1^8 \frac{x}{x}\, dx$	M1	1.1a	Use of integration by parts with 1 and $\ln x$	
				A1	1.1	May not include limits at this stage	
			$= [x\ln x - x]_1^8$	A1	2.5	All correct including limits	
			$= 8\ln 8 - 8 - (1\ln 1 - 1) = 8\ln 8 - 7$ **AG**	E1	2.1	Convincing completion	
				[4]			

Question			Answer	Marks	AOs	Guidance	
11			Area of rectangles $= \ln 2 + \ln 3 + \ldots + \ln 8$	**B1**	**1.1**		
			$= \ln(2 \times 3 \times \ldots \times 8)$	**M1**	**2.2a**		
			Area of rectangles $> \int_1^8 \ln x \, dx$, so $\ln 8! > 8\ln 8 - 7$ **AG**	**E1**	**2.1**	Convincing completion	
				[3]			
12	**(i)**	(A)	$f'(x) = \frac{1}{x}$ oe	**M1**	**1.2**	Correct derivative	
			$x > 0$ [throughout domain] so gradient is positive, i.e. $\ln x$ is increasing	**E1**	**2.1**	Convincing completion of argument	
				[2]			
		(B)	$\frac{d^2y}{dx^2} = -\frac{1}{x^2}$ oe	**M1**	**1.2**	Correct derivative	
			This is negative so curve is concave downwards	**E1**	**2.1**	Convincing completion of argument	
				[2]			
	(ii)		Since the curve is concave downwards, for each trapezium there is part of the area under the curve which is not included in the trapezium	**E1**	**2.4**	Or other convincing explanation; e.g. The shape of the curve means that all the trapeziums are below it	The explanation must refer to *all* the trapeziums
				[1]			

Question			Answer	Marks	AOs	Guidance	
13			$\dfrac{(2n)!}{n!n!} \approx \dfrac{\sqrt{4\pi n}\left(\dfrac{2n}{\mathrm{e}}\right)^{2n}}{\left(\sqrt{2\pi n}\right)^2\left(\dfrac{n}{\mathrm{e}}\right)^{2n}}$	**M1**	**1.1**	Use of Stirling's formula	
			$= \dfrac{2\sqrt{\pi n}}{2\pi n}\left(\dfrac{2n\mathrm{e}}{n\mathrm{e}}\right)^{2n}$	**M1**	**3.1a**	Correct simplification of either the power terms or the 'πn' terms	$\dfrac{1}{\sqrt{\pi n}} \times \dfrac{\left(\dfrac{2n}{\mathrm{e}}\right)^{2n}}{\left(\dfrac{n}{\mathrm{e}}\right)^{2n}}$
			$= \dfrac{2^{2n}}{\sqrt{\pi n}} = \dfrac{4^n}{\sqrt{\pi n}}$ **AG**	**A1**	**2.1**	Correct completion	
				[3]			