OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics

Practice Paper – Set 2

Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X=r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1-p$

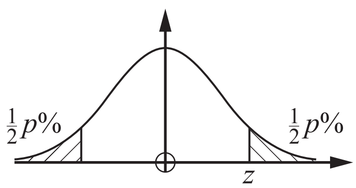
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u+at$

$s = ut+\frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2+2as$

$s = vt-\frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u}+\mathbf{a}t$

$\mathbf{s} = \mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t-\frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (22 marks)

1 The equation of a circle is

$$x^2 - 4x + y^2 + 6y - 12 = 0.$$

Find

(i) the coordinates of the centre of the circle, **[2]**

(ii) the radius of the circle. **[2]**

2 In a certain school there are 273 boys and 327 girls. A proportional stratified sample of 50 pupils is required.

How many girls should be in the sample? **[2]**

3 The probability distribution for the discrete random variable X is shown in Fig. 3.

x	0	1	2	3	4
$\mathrm{P}(X = x)$	$0.4a$	$0.5a$	$0.3a$	$0.2a$	$0.1a$

Fig. 3

(i) Calculate the value of the constant a. **[2]**

(ii) Calculate $\mathrm{P}(X \geqslant 2)$. **[2]**

(iii) Describe the shape of the distribution. **[1]**

4 Fig. 4 shows a cuboid $ABCDEFGH$. You are given that $FA > BC > AB > 0$. The distance between F and C is 5 cm. Given that the lengths AB and BC are both integers, prove that the length FA cannot be an integer. **[3]**

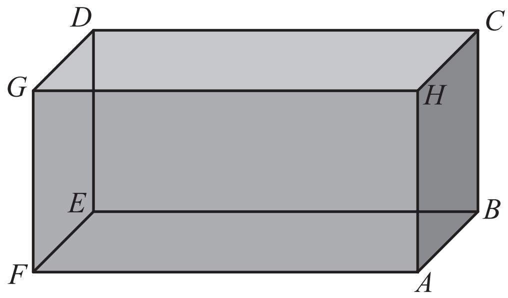


Fig. 4

5 Show that $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$ can be written in the form $a+\sqrt{b}$ where a and b are integers. **[3]**

6 **(i)** Show that $(x-3)$ is a factor of $\mathrm{f}(x) = 6x^3 - 17x^2 - 5x + 6$. **[1]**

(ii) Show that $\mathrm{f}(x)$ can be written as the product of 3 linear factors. **[4]**

Answer **all** the questions

Section B (78 marks)

7 In a game show, each contestant is asked a number of questions. Each question has a choice of three possible answers.

The probability that John knows the answer to any given question is $\frac{2}{5}$.

If John doesn't know the answer he guesses. The probability that he guesses correctly is $\frac{1}{3}$.

Calculate the probability that John answers two consecutive random questions correctly**.** **[4]**

8 Initially a car is travelling along a stretch of motorway at a steady speed of $22\,\mathrm{m\,s^{-1}}$. The driver changes lane and accelerates in order to overtake a lorry. The speed of the car, $V\,\mathrm{m\,s^{-1}}$, is modelled by the formula

$$V = A + B(1 - \mathrm{e}^{-0.2t}),$$

where A and B are constants to be determined and t is the time in seconds after the car has started to accelerate.

2 seconds after the car starts to accelerate it is travelling at $25.3\,\mathrm{m\,s^{-1}}$.

(i) Find the values of A and B, giving the value of B correct to 2 significant figures. **[3]**

5 seconds after the car starts to accelerate it is travelling at $28.4\,\mathrm{m\,s^{-1}}$.

(ii) Determine whether the model is a good fit for the data. **[2]**

(iii) According to the model, what is the acceleration of the car at $t = 5$? **[3]**

On this stretch of motorway the speed limit is $110\,\mathrm{km\,h^{-1}}$. The level of penalty given to a driver caught speeding above $110\,\mathrm{km\,h^{-1}}$ depends on the percentage the speed is above the speed limit. The driver continues to accelerate until reaching the maximum speed predicted by the model.

(iv) Determine whether the driver will exceed the speed limit by 10%. **[4]**

9 Nigel investigated migration in and out of different areas in the United Kingdom. He considered all the data available for long term international inflow and outflow rates per thousand resident population in 2012. Fig. 9.1 shows a scatter diagram for Nigel's sample after the data had been cleaned.

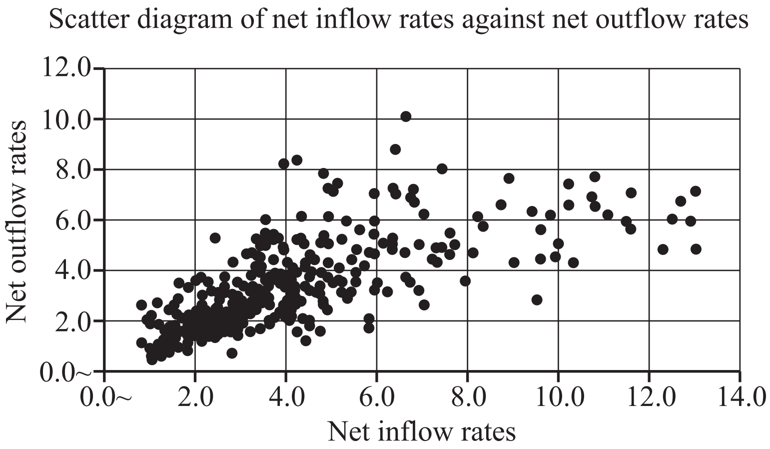


Fig. 9.1

Nigel used software to generate summary statistics. Fig. 9.2 shows summary statistics for inflow rates.

Statistics	
n	391
mean	6.8
s	8.579
$\sum x$	2644.9
$\sum x^2$	46598.17
min	0.8
Q_1	2.5
median	4.0
Q_3	6.8
max	91.8

Fig. 9.2

Nigel decided to clean the data by discarding all outliers, and so the pairs of values with long term inflow rates greater than 13.25 were discarded.

(i) Use information from Fig. 9.2 to show how Nigel calculated the value 13.25. **[1]**

Nigel then adopted a similar procedure by considering summary statistics for the long term net outflow rates. As a result he obtained a sample of size 334 drawn from a population of 391.

(ii) State **one** strength and **two** weaknesses of Nigel's method for selecting his sample. **[3]**

Nigel found that the product moment correlation coefficient for these data is 0.6916. He used an online statistics calculator to find the associated p-value. A screenshot of the result is shown in Fig. 9.3.

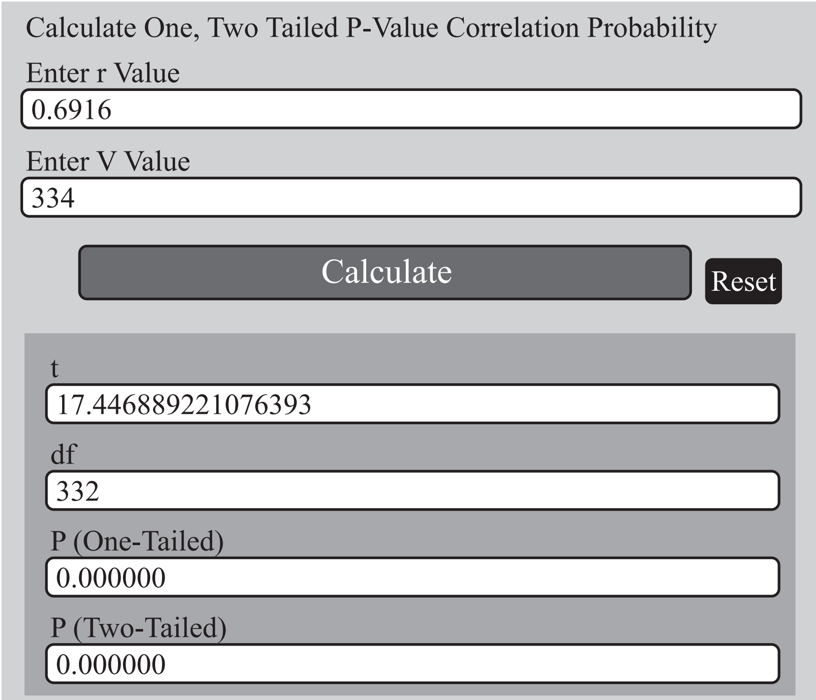


Fig. 9.3

Nigel made the following statement.

"My analysis proves that there is positive correlation between long term net inflow and long term net outflow rates per thousand resident population. Therefore high long term inflow rates cause high long term outflow rates."

(iii) Give **three** reasons why Nigel's statement is flawed. **[3]**

10 In this question you must show detailed reasoning.

Given that

$$(1+ax)^n = 1+6x-6x^2+\dots,$$

where a and n are constants, find the values of a and n. **[6]**

11 The pre-release data set consists of a subset of the data about countries from the CIA World Facebook.

Data for Croatia shows that the average number of mobile phone subscribers per 1000 population is 1111.72.

(i) Explain how this average can be greater than 1000. **[1]**

It is assumed that, once the data have been cleaned, the average number of phones per 1000 population in different countries can reasonably be approximated by a Normal distribution with mean 996 and standard deviation 407.

(ii) State two ways in which the data may have been cleaned. **[2]**

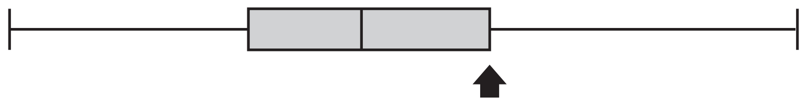


Fig. 11.1

Fig. 11.1 shows a box plot of the cleaned data.

(iii) Give the name of the measure which the arrow is pointing to. **[1]**

(iv) Use the approximate Normal distribution to estimate a value for this measure. **[2]**

Fatima thinks that the mean number of mobile phone subscribers per 1000 population for the countries of the world has increased since the data were collected. In order to test this he obtains some up-to-date data for a random sample of countries and uses software to conduct the appropriate hypothesis test at the 5% level of significance. The output from the software is shown in Fig. 11.2.

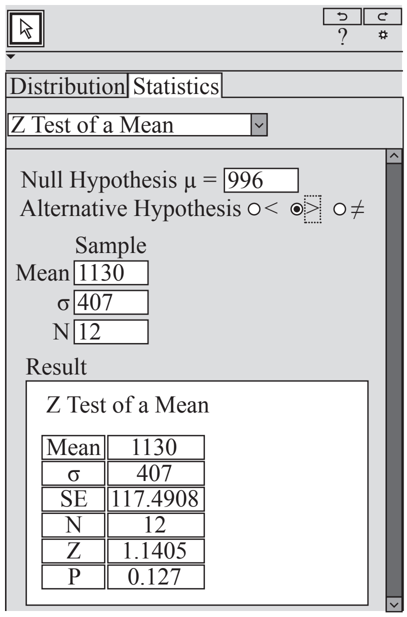


Fig. 11.2

(v) State the conclusion of Fatima's test, explaining your reasoning. **[4]**

(vi) With reference to the pre-release data set, comment on the sample size Fatima has chosen. **[1]**

12 A team of biologists are monitoring a subspecies of vole on a remote island. Previous studies have shown that the population mean mass of adult male voles is 26.2 g. It is believed that, due to changes in the environment, the mean mass may have altered. Three of the biologists conducted a new survey and each collected a random sample of adult male voles. The voles were weighed and tagged and then released unharmed.

Fig. 12 shows summary statistics for the 3 samples.

Statistics	Sample 1	Sample 2	Sample 3
Mean	22.6	22.8	23.7
Standard deviation	2.983287	3.258527	4.122196
Variance	8.9	10.618	16.9925
Minimum	19.2	18.6	18.4
Maximum	29.1	30.1	32
Q_1	20.3	20.1	19.8
Median	21.7	22.4	23.2
Q_3	23.95	24.3	26.1
Count	9	11	17

Fig. 12

(i) For each sample calculate $\sum x$ and $\sum x^2$. **[4]**

(ii) Use your answers to part **(i)** to calculate the mean and variance of the mass of all the voles caught in the survey. **[2]**

The biologists use the results of the survey to test, at the 5% level, the hypothesis $H_0: \mu = 26.1$ against the hypothesis $H_1: \mu \neq 26.1$.

(iii) Assuming that the distribution of the masses of adult male voles may be modelled by the Normal distribution with the value for the variance found in part **(ii)**, calculate the critical region for this test. **[3]**

(iv) State the conclusion reached by the biologists, explaining your answer. **[1]**

13 **In this question you must show detailed reasoning.**

The manufacturers of "Miracleroot" claim that, as long as their instructions are followed, 80% of cuttings taken from Christmas trees will grow roots and develop into healthy new trees. A horticulturist suspects that this claim is optimistic. He takes 500 cuttings, and after carefully following the manufacturer's instructions, finds that 380 of them developed roots and developed into healthy young trees.

Carry out a hypothesis test at the 1% level to determine whether there is any evidence to suggest that the manufacturer's claim is optimistic. **[7]**

14 In this question you must show detailed reasoning.

The probability that Judith has meat for her evening meal is 0.2 and the probability she has fish is 0.35; otherwise she has a vegetarian dish. If she has meat, the probability that she has indigestion is 0.8, if she has fish the probability that she has indigestion is 0.5 and if she has a vegetarian dish the probability that she has indigestion is 0.1.

Last night Judith had indigestion.

Calculate the probability that she had meat for her evening meal. **[6]**

15 Find $\int 4x^2 \sin 2x \, dx$. **[6]**

16 In this question you must show detailed reasoning.

The equation of a curve is $y = \dfrac{2x}{\ln x}$. Show that the only stationary point on this curve is a minimum point at (e, 2e). **[9]**

END OF QUESTION PAPER

BLANK PAGE