Mechanics Minor – Massive Past Paper Mix

From MEI papers M2 and M3 covering the period Jan 01 – Jan 03

Topics covered: Collisions, Centre of Mass, Moments, Dimensional Analysis and Work, Energy and Power

M2 Jan 01 Q1 - Collisions

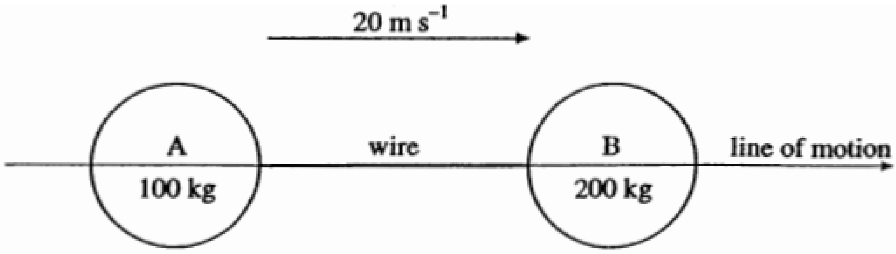

Fig. 1

Two small spacecraft, A with mass 100 kg and B with mass 200 kg, are modelled as moving in the absence of external forces. Both spacecraft are moving at 20 $m s^{-1}$ along the same line of motion. They are connected by a light wire in the line of motion, as shown in Fig. 1.

In an attempt to bring the spacecraft together, the tension in the wire is instantaneously increased so that an impulse of magnitude 1100 N s acts on each of the spacecraft.

(i) Show that the new velocity of A is 31 $m s^{-1}$ in the original direction of motion and find the new velocity of B. [3]

The spacecraft collide with a direct impact. The coefficient of restitution is $\frac{3}{4}$.

(ii) Show that the velocity of A after the collision is 11.75 $m s^{-1}$ in the original direction of motion. [6]

Before the wire becomes taut again, a component of spacecraft A is fired off in the opposite direction to the motion of A. The component has mass 20 kg. The main part of A and the component separate at 70 $m s^{-1}$.

(iii) Show that the new velocity of the component is 44.25 $m s^{-1}$ in the opposite direction to the motion of A. [4]

The two spacecraft now collide again and coalesce.

(iv) Calculate the final joint velocity of the two spacecraft. [2]

M2 Jan 01 Q2 – Centre of Mass

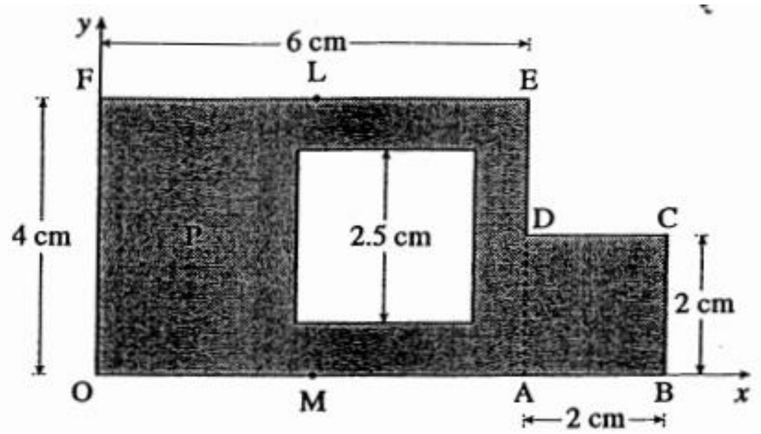

Fig. 2

In this question, coordinates refer to the axes shown in Fig. 2 and the units are centimetres. Answers should be given correct to three significant figures.

A thin, uniform sheet of metal is cut to form the shape shown in Fig. 2. OAEF is a rectangle and ABCD is a square with the dimensions shown. A square of side 2.5 cm and centre (4,2) has been removed leaving the shaded part P.

(i) Show that the x-coordinate of the centre of mass of P is 3.45, correct to three significant figures, and calculate the y-coordinate. [5]

L and M are the mid-points of EF and OA respectively. The shape P is freely suspended from L.

(ii) Calculate the angle that LM makes with the vertical. [3]

The mass of the square removed is 0.05 kg.

(iii) What vertical force must be applied at O so that when P is freely suspended from L, the line LM is vertical? [4]

The shape is now folded along AD so that the square ABCD is at right angles to OAEF. The x- and y-axes are along OA and OF, as before.

(iv) Calculate the x- and y-coordinates of the centre of mass of P now that it is bent. [3]

M2 Jan 01 Q3 - Moments

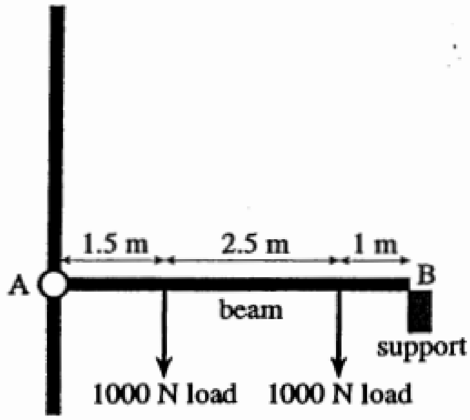

Fig. 3.1

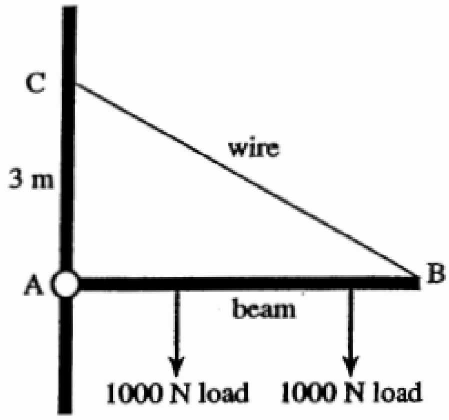

Fig. 3.2

Overhead cables for a tramway are supported by uniform, rigid, horizontal beams of weight 1500 N and length 5 m. Each beam, AB, is freely pivoted at one end A and supports two cables which may be modelled by vertical loads, each of 1000 N, one 1.5 m from A and the other at 1 m from B.

In one situation, the beam is held in equilibrium by resting on a small horizontal support at B, as shown in Fig. 3.1.

(i) Draw a diagram showing all the forces acting on the beam AB. Show that the vertical force acting on the beam at B is 1850 N. [4]

In another situation, the beam is supported by a wire, *instead of the support at* B. The wire is light, attached at one end to the beam at B and at the other to the point C which is 3 m vertically above A, as shown in Fig. 3.2.

(ii) Calculate the tension in the wire. [5]

(iii) Find the magnitude and direction of the force on the beam at A. [6]

M2 Jan 01 Q4 – Work, Energy and Power

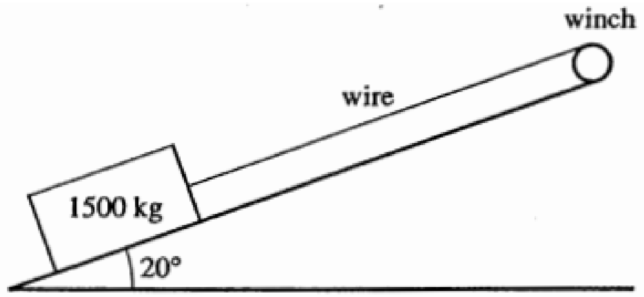

Fig. 4.1

A winch pulls a crate of mass 1500 kg up a slope at 20° to the horizontal. The light wire attached to the winch and the crate is parallel to the slope, as shown in Fig. 4.1.

The crate takes 50 seconds to move 25 m up the slope at a constant speed when the power supplied by the winch is 6 kW.

(i) How much work is done by the tension in the wire in the 50 seconds? [1]

(ii) Calculate the resistance to the motion of the crate up the slope. [4]

(iii) Show that the coefficient of friction between the crate and the slope is 0.50 (correct to two significant figures). [3]

The winch breaks down and the crate is then *pushed* up the slope by a mechanical shovel by means of a constant force of 16 000 N inclined at 15° to the slope, as shown in Fig. 4.2. You may assume that the crate does not tip up.

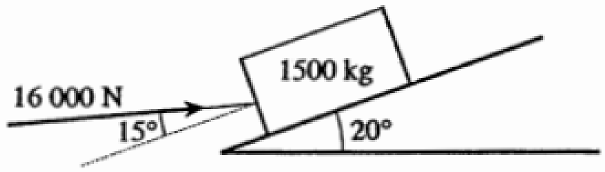

Fig. 4.2

(iv) Calculate the distance travelled by the crate up the slope as it speeds up from rest to 2.5 m s^{-1}. [You may assume the coefficient of friction between the crate and the slope is exactly 0.5.] [7]

M3 Jan 01 Q1 – Dimensional Analysis

An experiment consists of stretching a length of wire and fixing its ends. The wire is then plucked to produce transverse vibrations. The frequency of these vibrations, f, is modelled by the equation

$$f = kP^{\alpha} l^{\beta} z^{\gamma}$$

where P is the tension in the wire, l is the length of the stretched wire, z is the mass per unit length of the stretched wire and k is a dimensionless constant.

(i) Given that the dimensions of frequency are T^{-1}, use dimensional analysis to show that $\alpha = \frac{1}{2}$, $\beta = -1$ and $\gamma = -\frac{1}{2}$. [5]

Two pieces of wire, A and B, used in the experiment both have modulus of elasticity 500 N.

(ii) Piece A has natural length 0.9 m and stretched length 0.945 m. Calculate the tension in the wire. [2]

(iii) Piece B is stretched to a length of 0.81 m. The tension in the wire is 40 N. Calculate the natural length of the wire. [4]

(iv) *In their unstretched state*, the two wires had the same mass per unit length. In the experiment, wires A and B produce frequencies f_A and f_B respectively. Calculate the ratio $f_A : f_B$. [4]

M2 Jun 01 Q1 - Collisions

Two young skaters, Percy of mass 55 kg and Queenie of mass 45 kg, are moving on a smooth horizontal plane of ice. You may assume that there are no external forces acting on the skaters in this plane. Percy and Queenie are moving with speeds of $2\ \text{m s}^{-1}$ and $\frac{4}{3}\ \text{m s}^{-1}$ respectively **towards** one another in the same line of motion. When they meet they embrace.

(i) Calculate the common velocity of the two skaters after they meet and the magnitude of the impulse between them in the collision. [5]

Percy and Queenie, still together, collide directly with a moving skater, Roger, of mass 60 kg. The coefficient of restitution in the collision is 0.2. After the collision, Percy and Queenie have a speed of $0.1\ \text{m s}^{-1}$ in the same direction as before the collision.

(ii) Calculate Roger's velocity before the collision and his velocity after it. [6]

M2 Jun 01 Q2 – Work, Energy and Power

A parcel of mass 20 kg slides down a slope at 35° to the horizontal. Its speed increases from 4 m s^{-1} to 6 m s^{-1} while sliding a distance of 5 m down the slope.

(i) Calculate the work done against the resistance to motion. [4]

(ii) Assuming that a constant frictional force between the parcel and the slope is the only resistance to motion, show that the coefficient of friction between the parcel and the slope is 0.45, correct to two significant figures. [4]

(iii) For what value of the coefficient of friction would the parcel slide down the slope at a constant speed? [2]

The parcel is sliding down the slope and the coefficient of friction is 0.45. A force, applied parallel to the slope, does 520 J of work and brings the parcel to rest from 6 m s^{-1} in x m.

(iv) Calculate the value of x. [5]

M2 Jun 01 Q3 – Centre of Mass and Moments

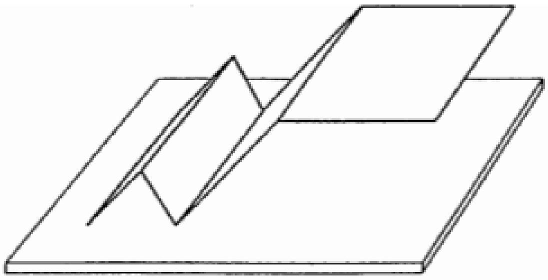

Fig. 3.1

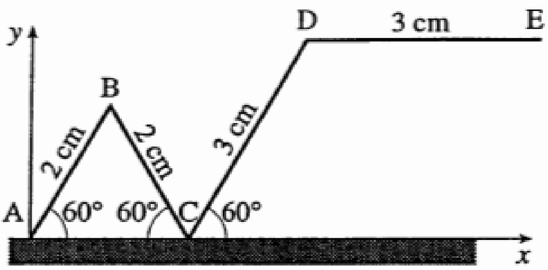

Fig. 3.2

A uniform, rectangular lamina of mass 25 kg is folded and placed on a horizontal floor, as shown in Fig. 3.1. Fig. 3.2 shows the cross-section ABCDE of the folded lamina. The dimensions and angles of the cross-section are given in Fig. 3.2 and DE is horizontal.

(i) Show that the x-coordinate of the centre of mass of the lamina is 2.725, referred to the axes shown in Fig. 3.2. Calculate also the y-coordinate, referred to the same axes, giving your answer correct to three decimal places. [6]

(ii) Explain briefly why the lamina cannot be in equilibrium in the position shown without the application of an additional force. [2]

(iii) What is the least vertical force that must be applied to the lamina at A so that it will stay in equilibrium in the position shown? [4]

Instead of applying the vertical force at A, a horizontal force is applied to the lamina at E.

(iv) Assuming that the lamina does not slide on the floor, calculate the greatest value the horizontal force at E can take without the lamina turning *anti*-clockwise. [3]

M2 Jan 02 Q1 – Work, Energy and Power

A tractor and its plough have a combined mass of 6000 kg. When developing a power of 5 kW, the tractor is travelling at a steady speed of 2.5 $m s^{-1}$ along a horizontal field.

(i) Calculate the resistance to the motion. [2]

The tractor comes to a patch of wet ground where the resistance to motion is different. The power developed by the tractor during the next 10 seconds has an average value of 8 kW over this time. During this time, the tractor accelerates uniformly from 2.5 $m s^{-1}$ to 3 $m s^{-1}$.

(ii) Show that the work done against the resistance to motion during the 10 seconds is 71 750 J.

Assuming that the resistance to the motion is constant, calculate its value. [7]

The tractor now comes to a slope at $\sin^{-1}(\frac{1}{20})$ to the horizontal. The non-gravitational resistance to motion on this slope is 2000 N. The tractor accelerates uniformly from 3 $m s^{-1}$ to 3.25 $m s^{-1}$ over a distance of 100 m while climbing the slope.

(iii) Calculate the time taken to travel this distance of 100 m and the average power required over this time period. [6]

[Total 15]

M2 Jun 02 Q1 - Collisions

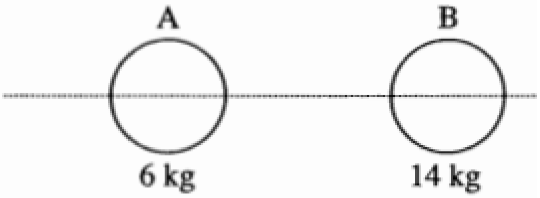

Fig. 1

Two circular discs slide on a smooth horizontal surface. Disc A has mass 6 kg and disc B has mass 14 kg and both are initially at rest. A force of 12 N acts on disc A for 4 seconds and this disc then collides directly with disc B. The coefficient of restitution between the two discs is 0.25.

(i) Calculate the velocity of disc A before the collision. [2]

(ii) Show that, after the collision, disc B has speed $3\,\mathrm{m\,s^{-1}}$ and find the new speed of disc A. [6]

(iii) Calculate the impulse on disc A in this collision. [2]

M2 Jun 02 Q2 – Work, Energy and Power

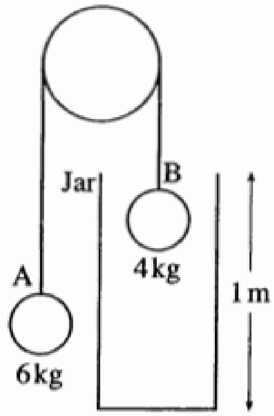

Fig. 2

Two objects A and B are connected by a light, inextensible string which passes over a smooth fixed pulley. The mass of A is 6kg and of B is 4kg.

The object B is inside a fixed empty jar 1 m tall. At no time does B reach the pulley or touch the sides of the jar.

Object B is held at rest and then the system is released. Object A falls a distance of 0.8 m.

(i) Calculate the change in gravitational potential energy of the system. [2]

(ii) Calculate the speed of the objects when A has fallen the 0.8 m. [3]

The jar is now filled with liquid which may be assumed to provide a constant resistance of 12N to the motion of B. Object B is initially held at rest at the bottom of the jar and then the system is released. When A has dropped 0.8 m, the string breaks at B.

(iii) Calculate

(*A*) the kinetic energy of B when the string breaks, [3]

(*B*) the distance B rises after the string breaks, [3]

(*C*) the speed of B when it reaches the bottom of the jar. [3]

[Total 14]

M2 Jun 02 Q3 – Centre of Mass

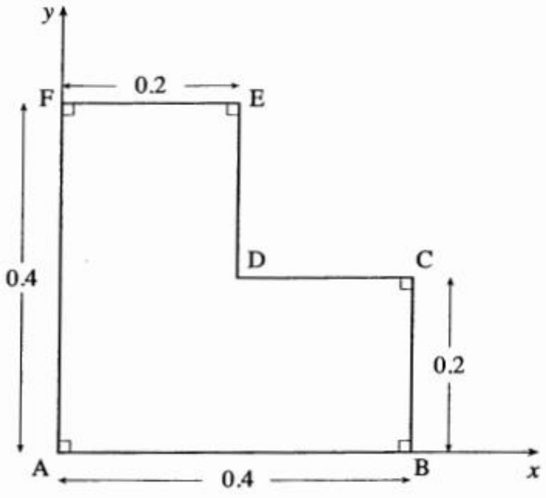

Fig. 3.1

In this question, the units on the axes are metres. Answers should be given in exact numbers or correct to three significant figures. Coordinates are referred to the axes shown in Fig. 3.1.

Fig. 3.1 shows a shape ABCDEF with the dimensions given.

(i) Regarding ABCDEF as a uniform lamina, show that the coordinates of its centre of mass are $(\frac{1}{6}, \frac{1}{6})$. [5]

(ii) Now regarding ABCDEF as a framework of uniform rods, show that the coordinates of its centre of mass are (0.175, 0.175). [5]

A toolbox is shown in Fig. 3.2. This toolbox is a hollow prism with ends ABCDEF and GHIJKL perpendicular to its long axis. The toolbox is 0.6 m long and is made from thin, uniform material.

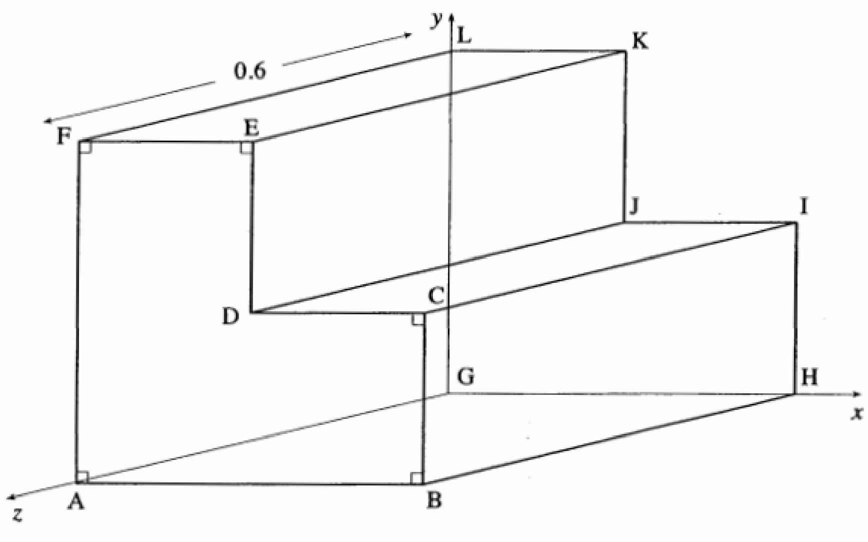

Fig. 3.2

(iii) By using your answers from parts **(i)** and **(ii)**, or otherwise, calculate the position of the centre of mass of the toolbox, referred to the axes shown in Fig. 3.2. [6]

[Total 16]

M2 Jun 02 Q4 - Moments

(a) A uniform rod AB of length 8 m and weight 180 N is held in horizontal equilibrium by two vertical wires. One wire is 1 m from A and the other 2 m from B, as shown in Fig. 4.1.

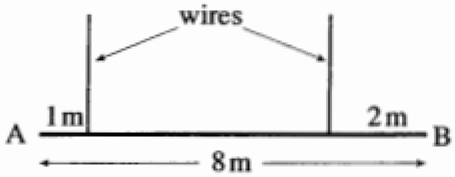

Fig. 4.1

(i) Draw a diagram showing all the forces acting on the rod. [1]

(ii) Calculate the tensions in the wires. [4]

(b)

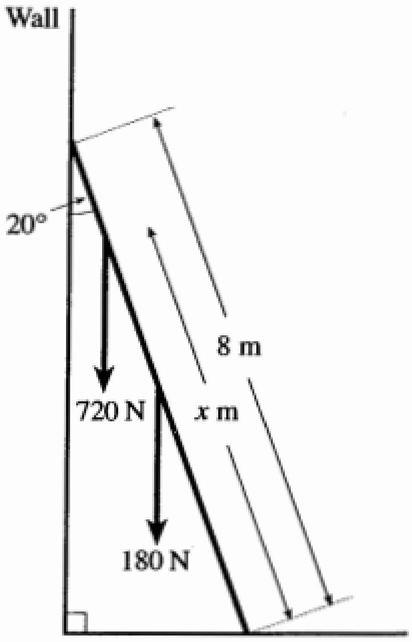

Fig. 4.2

A uniform ladder of length 8 m and weight 180 N rests against a smooth, vertical wall and stands on a rough, horizontal surface. A woman of weight 720 N stands on the ladder so that her weight acts at a distance x m from its lower end, as shown in Fig. 4.2.

The system is in equilibrium with the ladder at 20° to the vertical.

(i) Show that the frictional force between the ladder and the horizontal surface is F N, where

$$F = 90(1 + x)\tan 20°.$$ [5]

(ii) Deduce that F increases as x increases and hence find the values of the coefficient of friction between the ladder and the surface for which the woman can stand anywhere on the ladder without it slipping. [5]

[Total 15]

M3 Jun 02 Q1 – Dimensional Analysis

(i) Write down the dimensions of velocity, acceleration and force. [2]

(ii) Use the definitions of work, kinetic energy and change in gravitational potential energy to show that these quantities have the same dimensions. [3]

The tension in a stretched wire is given by $T = \frac{YAx}{l_0}$, where A is the cross-sectional area of the wire, l_0 is the natural length of the wire, x is the extension and Y is a quantity called Young's modulus which depends on the material from which the wire is made.

(iii) Determine the dimensions of Young's modulus. [3]

The energy stored in the stretched wire is given by the equation

$$E = cY^{\alpha}\left(\frac{A}{l_0}\right)^{\beta} x^{\gamma}$$

where c is a dimensionless constant.

(iv) Use dimensional analysis to determine the value of α and to find a relationship between β and γ. [4]

M2 Jan 01 Q2 – Centre of Mass and Moments

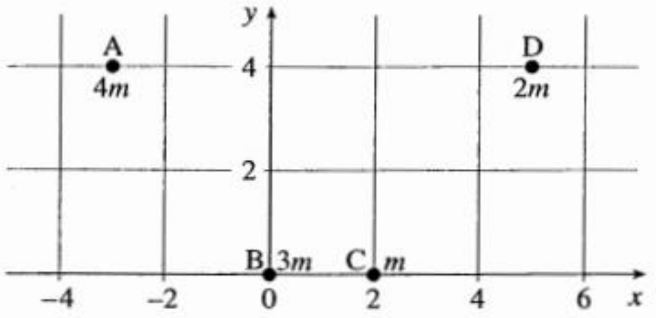

Fig. 2.1

In this question, coordinates are referred to the axes shown in Fig. 2.1.

Four particles at A, B, C and D with masses $4m$, $3m$, m and $2m$ lie in a plane with positions $(-3, 4)$, $(0, 0)$, $(2, 0)$ and $(5, 4)$, respectively.

(i) Calculate the coordinates of the centre of mass of the four particles. [5]

A thin, uniform, rigid wire of mass $12m$ connects A to B, B to C and C to D, as shown in Fig. 2.2.

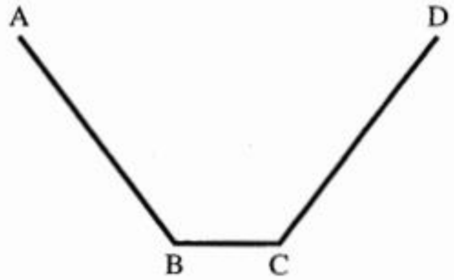

Fig. 2.2

(ii) Calculate the coordinates of the centre of mass of the wire. [5]

(iii) Calculate the coordinates of the combined centre of mass of the wire and the particles at A, B, C and D. [3]

A is connected to D by means of a further straight rigid wire of negligible mass. The combined system of wires and the particles at A, B, C and D is suspended freely from the mid-point of AD.

(iv) What extra mass must be added at C if the system is to hang in equilibrium with AD horizontal? [3]

[Total 16]

M2 Jan 02 Q3 – Work, Energy and Power

A car of mass 800 kg is travelling along a straight, level road.

The car is travelling at a steady speed of $25\ \mathrm{m\,s^{-1}}$ when the engine is developing 32 kW of power.

(i) Calculate the resistance to the motion of the car.

Assuming that the resistance to the car's motion is constant, how much work is done against it as the car travels 100 m? [4]

The power developed by the engine is increased so that it averages 45 kW over the next 10 seconds. At the end of this time the car is travelling at a speed of $v\ \mathrm{m\,s^{-1}}$. The total work done against the resistance to motion during the 10 seconds is 340 kJ.

(ii) Show that $v = 30$. [6]

With the engine developing no power, the speed of the car decreases from $35\ \mathrm{m\,s^{-1}}$ to $15\ \mathrm{m\,s^{-1}}$ as it travels 200 m *up* a slope at an angle of θ to the horizontal, where $\sin\theta = \frac{1}{14}$, as shown in Fig. 3.

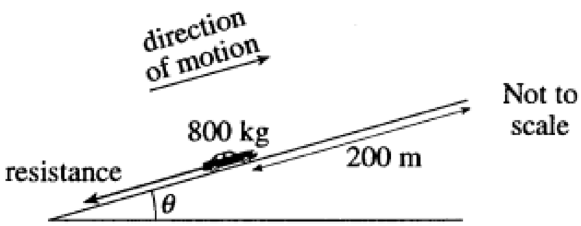

Fig. 3

(iii) How much work is done against the resistance to motion over this 200 m? [5]

[Total 15]

M2 Jan 03 Q4 - Moments

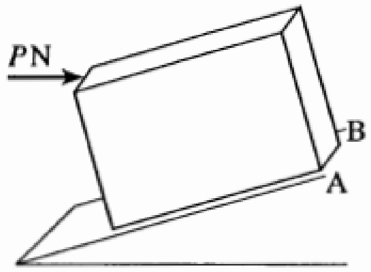

Fig. 4.1

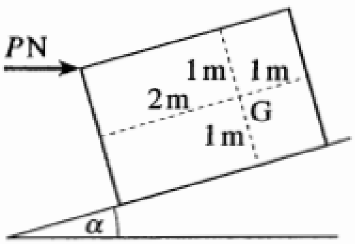

Fig. 4.2

A packing case in the shape of a cuboid is on a rough plane inclined at an angle α to the horizontal. The packing case is being pushed by a **horizontal** force of P N applied perpendicular to and in the centre of an edge of the case, as shown in Fig. 4.1. Fig. 4.2 is a side elevation showing the dimensions of the packing case and the position of G, the centre of mass of the packing case and its contents.

The weight of the packing case and contents is 840 N, $\sin\alpha = \frac{7}{25}$, $\cos\alpha = \frac{24}{25}$ and the coefficient of friction between the packing case and the plane is μ.

(i) Initially $P = 0$ and the packing case is in equilibrium. Show that $\mu \geqslant \frac{7}{24}$. [4]

(ii) Subsequently $P > 0$. Write down the components of P parallel to and perpendicular to the plane. Show that the moment of the pushing force about the edge AB, shown in Fig. 4.1, is $\frac{27}{25}P$ N m clockwise. [4]

(iii) The value of P is such that the packing case is in equilibrium but about to turn about the edge AB.

Draw a diagram showing all of the forces acting on the packing case.

Show that $P = 964$, correct to three significant figures. [6]

[Total 14]