Mech Major – Mark Scheme

1.

1(i)	$2g = 400x$	M1	use of Hooke's law and equilibrium	
	$x = 0.049$	A1		
				2
(ii)	$0.5 - e$	B1	accept unsimplified	
				1
(iii)	$T_1 = T_2 + mg$	M1	equilibrium equation	
	$400e = 400(0.5 - e) + 2g$	A1	allow their (ii) instead of $(0.5 - e)$	
	$800e = 219.6$	M1	solving an equation with more than one term in e	
	$e = 0.2745$	A1	cao	
				4
(iv)	$T_1 = 400e = 109.8$	F1	400 times their e	
	$T_2 = 400(0.5 - e) = 90.2$	F1	or $T_1 - 19.6$	
				2
(v)	(let y be displacement below equilibrium position)			
	$m\ddot{y} = T_2 + mg - T_1$	M1	N2L in general position	
		B1	weight term (ignore sign)	
	$= 400(0.5 - e - y) + mg - 400(e + y)$	M1	two distinct tensions in terms of y (or equivalent)	
		A1	all correct	
	$\Rightarrow \ddot{y} = -400y$, i.e. SHM	E1	must be completely correct and conclude SHM	
	period $= \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{400}} = \frac{1}{10}\pi$	E1	dependent on both M1 marks	
				6

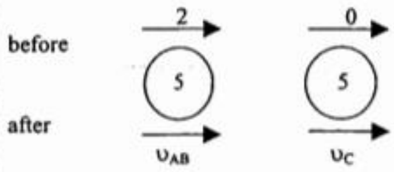
2.

3(i)	$v^2 = 2 \times 9.8 \times 62.5 \Rightarrow v = 35\,\text{m s}^{-1}$	E1	using energy or constant acceleration formula(e)	
				1
(ii)	$\frac{1}{2}mv^2 = mg(62.5 + 25\sin\theta)$	M1	using energy	
	or $\frac{1}{2}mv^2 - mg \cdot 25\sin\theta = \frac{1}{2}m \cdot 35^2$	A1	$25mg\sin\theta$ (or equivalent) seen	
		A1	correct equation	
	$\Rightarrow v^2 = 125g + 50g\sin\theta$	A1	or equivalent	
	$R - mg\sin\theta = \frac{mv^2}{25}$	M1	N2L with mv^2/r	
		A1		
	$R = \frac{m}{25}(125g + 50g\sin\theta) + mg\sin\theta$	M1	substitute their v^2 (but must be in terms of θ)	
	$R = mg(3\sin\theta + 5)$	E1		
				8
(iii)	radial $= \frac{v^2}{25} = 5g + 2g\sin\theta$	B1	follow their v^2 (but must be in terms of θ)	
	tangential $m\dot{v} = mg\cos\theta$	M1	N2L with cpt. of weight and no other force	
	$\dot{v} = g\cos\theta$	A1	ignore sign	
	$\lvert acc. \rvert = \sqrt{(5g + 2g\sin\theta)^2 + (g\cos\theta)^2}$	M1	finding magnitude	
	$= g\sqrt{25 + 20\sin\theta + 4\sin^2\theta + 1 - \sin^2\theta}$	M1	using $\cos^2\theta = 1 - \sin^2\theta$	
	$= g\sqrt{26 + 20\sin\theta + 3\sin^2\theta}$	E1		
				6

3.

4(i)	$\bar{x} = 0$ (symmetry)	B1	need justification
	$\bar{y} = \dfrac{\int x^2 y\,dy}{\int x^2\,dy}$	B1	formula
	$= \dfrac{\int_0^1 (y - 2y^3 + y^5)dy}{\int_0^1 (1 - 2y^2 + y^4)dy}$	M1	substitute & expand (both) – must have more than two terms for each
	$= \dfrac{\left[\frac{1}{2}y^2 - \frac{1}{2}y^4 + \frac{1}{6}y^6\right]_0^1}{\left[y - \frac{2}{3}y^3 + \frac{1}{5}y^5\right]_0^1}$	M1 A1 A1	integrate (either) – must have more than one term numerator denominator
	$= \dfrac{1/6}{8/15} = \frac{5}{16}$	E1	
			7
(ii)	$\dfrac{\lambda m \cdot \frac{5}{8} + m \cdot \frac{21}{16}}{\lambda m + m}$	M1 A1 A1	$\Sigma mx / \Sigma m$ or moments numerator denominator
	$= \dfrac{10\lambda + 21}{16(\lambda + 1)}$	E1	
			4
(iii)	need centre of mass in hemisphere	B1	may be implied
	$\dfrac{10\lambda + 21}{16(\lambda + 1)} < 1$	B1 M1	inequality solving (accept equation)
	$\lambda > \frac{5}{6}$	A1	cao (if from equation, > must be justified)
			4

4.

(i)	$10 = 2v$ so $v = 5$ so 5 m s^{-1}	B1		1
(ii)	$10 = (2+3)V$	M1	PCLM. Must deal with coalescence.	
	so $V = 2$ so 2 m s^{-1}	E1		2
(iii)				
	$10 = 5v_C + 5v_{AB}$	M1 A1	PCLM Any form	
	$\frac{v_C - v_{AB}}{0-2} = -0.6$	M1	NEL	
		A1	Any form	
	Solving $v_C = 1.6$ so 1.6 m s^{-1}	E1	Must be clearly shown. (Either solved or v_C subst in one equation to get v_{AB} and both values checked in the other)	5
(iv)	NEL applied perpendicular to the floor	M1	May be implied	
	Vert cpt $\downarrow$ $\sqrt{6g}$	B1	May be implied	
	After bounce $\uparrow$ $\sqrt{6g} \times 0.6 = \sqrt{2.16g}$	E1	If decimal equivalence used need at least 1 d.p.	
	Linear momentum conserved parallel to floor	M1	May be implied by using 1.6 after the bounce	
	Angle with the floor is $\arctan \frac{\sqrt{6g} \times 0.6}{1.6}$	M1	Use of arctan or equiv with numerator **their** $\sqrt{2.16g}$	
	$= 70.8243\ldots$ so $70.8°$ (3 s. f.)	A1	cao	6
			total	14

5.

1(i)	$[E](=[mgh\cos\theta]) = \mathrm{ML^2T^{-2}}$	B1	stated or used correctly	
	$[\omega] = \mathrm{T^{-1}}$	B1	stated or used correctly	
	$[I] = [E]/[\omega^2]$	M1		
	$= \mathrm{ML^2}$	A1	from equation	
				4
(ii)	$\mathrm{T} = (\mathrm{ML^2})^\alpha (\mathrm{MLT^{-2}})^\beta \mathrm{L}^\gamma$	M1	substitute dimensions (must be shown)	
	$\alpha + \beta = 0$	M1	compare indices for one dimension	
	$2\alpha + \beta + \gamma = 0$	M1	compare indices for all dimensions	
	$-2\beta = 1$			
	$\beta = -\frac{1}{2}$	A1	one correct	
	$\alpha = \frac{1}{2}, \gamma = -\frac{1}{2}$	A1	all correct	
			if no comparison then M1 for substituting, A2 for one correct, A2 for all correct	
				5
(iii)	θ small $\Rightarrow \sin\theta \approx \theta$	B1		
	$\Rightarrow \ddot{\theta} \approx -9\theta \Rightarrow$ SHM	E1	must conclude SHM	
				2
(iv)	$\theta = 0.05\sin(3t + \varepsilon)$	M1	or equivalent (accept non-zero value for ε)	
	$0.025 = 0.05\sin\varepsilon \Rightarrow \varepsilon = \frac{1}{6}\pi$	M1	calculate ε	
	$\theta = 0.05\sin(3t + \frac{1}{6}\pi)$ or $\theta = 0.05\cos(3t - \frac{1}{3}\pi)$	A1		
	$\theta = 0 \Rightarrow 3t + \frac{1}{6}\pi = \pi \Rightarrow t = \frac{5}{18}\pi$	B1		
				4

6.

2(i)	N2L towards centre: Force on engine $= 50000\dfrac{30^2}{500}$	M1	use of v^2/r	
		A1	identify as N2L	
		A1	all correct	
	N3L: Force on rails $=$ Force on engine $= 90000$	B1	identify both forces and use of N3L	
				4
(ii)	$50000 = 50000\dfrac{V^2}{500}$	M1	N2L with v^2/r	
	$\Rightarrow V = 10\sqrt{5} \approx 22.4\ \mathrm{m\,s^{-1}}$	A1		
				2
(iii)	component of acceleration $= \dfrac{30^2}{500}\cos\theta$	B1		
	$T + 50000g\sin\theta = 50000\dfrac{30^2}{500}\cos\theta$	B1	resultant force down bank	
		M1	N2L with component of acceleration	
		A1	correct equation (not $T = \ldots$)	
	$T = 90000\cos\theta - 490000\sin\theta$	E1	must follow from correct N2L equation	
	Alternative:			
	$T\cos\theta + R\sin\theta = 50000\dfrac{30^2}{500}$	B1		
	$R\cos\theta = T\sin\theta + 50000g$	B1		
	eliminate R	M1		
	$T = 90000\cos\theta - 490000\sin\theta$	E1		
				5
(iv)	$R = \sqrt{90000^2 + 490000^2} \approx 498200$	B1		
	$\alpha = \tan^{-1}(490000/90000) \approx 79.59$	M1		
	$R\cos(\theta + \alpha) = 50000 \Rightarrow \theta + \alpha = \cos^{-1}(50000/R) = 84.24$	M1	solving	
	$\Rightarrow \theta \approx 4.65°$	A1		
				4

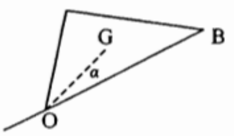
7.

3(i)	$0.2g = 5x/0.8$	M1	use of Hooke's law
	$x = 0.3136$	A1	
	length = 1.11(36) m	A1	
			3
(ii)	$\frac{1}{2}0.2v^2 + 0.2g \times 2.4 = \dfrac{5 \times 1.6^2}{2 \times 0.8}$	M1	use of energy
		A1	EPE term
		A1	correct equation
	$\Rightarrow v \approx 5.74 \text{ m s}^{-1}$	A1	
			4
(iii)	$\frac{1}{2}0.2v^2 + 0.2g \times 2.4 + \dfrac{5(2.4 - l_1)^2}{2l_1} = \dfrac{5 \times 1.6^2}{2 \times 0.8}$	M1	attempt EPE in terms of l_1
		A1	correct EPE in terms of l_1
		M1	energy equation (everything considered)
		A1	all correct
	$v^2 \geq 0 \Rightarrow$	M1	use of $v^2 \geq 0$ (accept =)
	$l_1^2 - 6.1184l_1 + 5.76 \leq 0$	A1	equation or inequality (or multiple)
	$\Rightarrow 1.162 \leq l_1 \leq 4.956$	M1	solving quadratic
	$l_1 \geq 1.162$	A1	inequality must be justified by algebra or explanation
			8

8.

4(i)	$y = \dfrac{h}{b}x$	B1	
	$\frac{1}{2}bh\bar{y} = \int_0^b \frac{1}{2}\left(\frac{h}{b}x\right)^2 dx$	B1	formula stated or used correctly
	$= \dfrac{h^2}{2b^2}\left[\frac{1}{3}x^3\right]_0^b = \frac{1}{6}h^2b$	M1	substitute and integrate RHS
		M1	use correct limits and divide by area
	$\bar{y} = \frac{1}{3}h$	E1	
	$\bar{x} = \frac{2}{3}b$	B1	
			6
(ii)	shear $\Rightarrow$ mass distribution from x-axis unchanged	M1	or calculation
	$\Rightarrow \bar{y} = \frac{1}{3}h$	E1	
	$\frac{1}{2}bh\bar{x} = \frac{1}{2}ch \cdot \frac{2}{3}c + \frac{1}{2}(b-c)h \cdot (c + \frac{1}{3}(b-c))$	M1	reasonable attempt at $\bar{x}$
		A1	correct equation
	$\bar{x} = \frac{1}{3}b + \frac{1}{3}c$	E1	
			5

(iii)

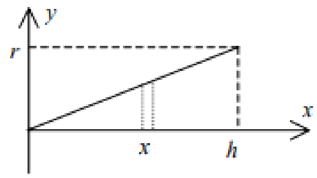


	topples $\Leftrightarrow \alpha > 60°$	B1	(accept =)
	($c > 0 \Rightarrow \alpha$ acute) so topples $\Leftrightarrow \tan\alpha > \sqrt{3}$	M1	(accept =)
	$\Leftrightarrow \dfrac{\frac{1}{3}h}{\frac{1}{3}b + \frac{1}{3}c} > \sqrt{3} \Leftrightarrow c < \dfrac{h}{\sqrt{3}} - b$	A1	inequality must be justified by algebra or explanation
	as $c > 0,\ h < \sqrt{3}b \Rightarrow c > \dfrac{h}{\sqrt{3}} - b \Rightarrow$ does not topple	E1	
			4

9.

(i)	$0.5mu^2 = 0.5mv^2 + mga(1-\cos\theta)$	M1 B1	Attempt to use KE + PE = const $mga(1-\cos\theta)$	
	so $v^2 = u^2 - 2ag(1-\cos\theta)$	E1	Clearly shown. Accept consistent signs throughout.	3
(ii)	$T - mg\cos\theta = \dfrac{mv^2}{a}$ $\left(= ma\omega^2\right)$	M1 B1 B1	Radial EoM attempted. Accept mg not resolved. All terms present. Do not accept T resolved. LHS RHS accept $u \leftrightarrow v$ and $a \leftrightarrow r$	
	so $T = mg\cos\theta + \dfrac{m}{a}\left(u^2 - 2ag\left(1-\cos\theta\right)\right)$	B1	Subst correct v^2 into **their** T.	
	$= \dfrac{m}{a}\left(3ag\cos\theta + u^2 - 2ag\right)$	A1	Accept any form with T the subject.	5
(iii)	Need $\left(3ag\cos\theta + u^2 - 2ag\right) \geq 0$ (for all θ)	M1	Accept =, > and given in words. Or start again.	
	Worst case is when $\theta = \pi$	B1	May be implied	
	giving $u^2 \geq 5ag$	E1	> must be justified	3
(iv)	$v = 0$ when $\theta = \dfrac{\pi}{2} \Rightarrow u^2 = 2ag$	M1 A1	Attempt to find u from initial conditions.	
	we need $2ag = 5(a-b)g$	M1	Attempt to relate result in (iii) to this situation. Accept $u^2 = 5bg$. Or start again.	
	so $5b = 3a$ or $b = \dfrac{3a}{5}$	A1		4
			total	15

10.

(i)	Vol is $\pi\int_{-1}^{1} x^2\,dy$			
	$= \pi\int_{-1}^{1}\left(2y^2 - y^4 + 1\right)dy$	M1	Limits not required	
	$= \pi\left[\frac{2y^3}{3} - \frac{y^5}{5} + y\right]_{-1}^{1}$	A1	Limits not required	
		M1	Limits dealt with correctly	
	$= 2\pi\left(\frac{2}{3} - \frac{1}{5} + 1\right) \;=\; \frac{44\pi}{15}$ giving $\frac{44\pi}{15}\ \text{m}^3$	E1	Clearly shown	
	$2(-y)^2 - (-y)^4 + 1 = 2y^2 - y^4 + 1$ so symmetrical about $y = 0$ so c. m. is at (0, 0)	B1	Must show symmetrical as well as give c.m.	5
(ii)	$\frac{1}{3}\pi r^2 h\bar{x} = \pi\int_0^h x\left(\frac{rx}{h}\right)^2 dx$	M1	Use of elementary discs radius rx/h or equivalent. Award for attempt to find $\int \pi x y^2 dx$ or equivalent	
		B1	Correct relation between x and y.	
	$= \left[\frac{\pi r^2}{h^2}.\frac{x^4}{4}\right]_0^h$	F1	Integration of **their** expression correct if at least quadratic. Neglect limits.	
	so $\frac{1}{3}\pi r^2 h\bar{x} = \frac{\pi r^2 h^2}{4}$	M1	Appropriate limits substituted and attempt to use M	
	$\bar{x} = \frac{3}{4}h$ so $\frac{1}{4}h$ from base.	E1	Clearly established	5
(iii)	Radius of cone is $\sqrt{2-1+1} = \sqrt{2}$	B1		
	Require $\frac{1}{3}\pi \times 2 \times h \times \frac{h}{4} = \frac{44\pi}{15}\times 1$	M1	Attempt to write equation using , e.g.,moments	
		A1	LHS or equivalent. FT **their** r.	
		A1	RHS or equivalent. FT **their** r.	
			[If three terms, A2 -1 each incorrect term. FT **their** r.]	
	$h = \sqrt{\frac{88}{5}}$ (= 4.1952…)	A1		5
			total	15

11.

(i)	$mg = 19.6 \times 0.05$	M1	Use of HL	
	$m = 0.1$	A1		2
(ii)	N2L $\downarrow$ $mg - F = m\ddot{x}$ $0.1g - 19.6(0.05 + x) = 0.1\ddot{x}$	M1 M1 A1 B1	Use of N2L. Condone sign errors and mg omitted. Linear expression for F in terms of x $\|F\|$ correct All correct including signs.	
	so $\ddot{x} + 196x = 0$	E1	[Award up to 4/5 if x taken +ve upwards]	5
(iii)	**either** $v^2 = \omega^2(a^2 - x^2)$	M1	Award M1 even if $x = 0$ used below	
	so $v^2 = 196(0.1^2 - (-0.05)^2) = 1.47$	A1	Correct substitution	
	so $v = 1.2124\ldots$ so 1.21 m s^{-1} (3 s. f.)	A1	cao	
	or $\frac{1}{2} \times 0.1 \times v^2 + 0.1 \times 9.8 \times 0.15 = \frac{1}{2} \times 19.6 \times 0.15^2$	M1	Equating KE + GPE to EPE	
	so $v = 1.2124\ldots$ so 1.21 m s^{-1} (3 s. f.)	A1 A1	All correct cao	
	or First find $x = 0.1\cos 14t$ (see below) $v = -1.4\sin 14t$	B1		
	sub $t = \frac{\pi}{21}$ (see below)	M1		
	$v = -1.2124\ldots$ and speed is 1.21 m s^{-1} (3 s. f.)	A1	cao	
	$x = 0.1\cos 14t$	B1	Obtaining expression for x in terms of t	
	we need $-0.05 = 0.1\cos 14t$	M1	Equating x in terms of t to -0.05	
	so $t = \frac{\pi}{21}$ (0.150 s (3 s. f.))	A1	cao. [Award 0 for ¼ period]	6
(iv)	**either** The motion is SHM, $a = 0.2$, $\omega = 14$ $v = v_{max}$ of the SHM $v_{max} = a\omega = 0.2 \times 14 = 2.8$	M1 B1 A1	May be implied May be implied	
	or $0.5 \times 0.1 \times v^2 + 0.1g \times 0.2 = \frac{19.6}{2}(0.25^2 - 0.05^2)$	M1 M1	Equating KE + GPE to EPE Correct change in EPE	
	$v = 2.8$	A1	[SC1 for $0.5 \times 0.1 \times v^2 = 0.5 \times k \times 0.2^2$]	3
			total	16

12.

1(i)	$T_{AB} = \dfrac{20\times0.5}{0.5} = 20\text{ N}$	M1 A1	Hooke's law
	$T_{BC} = \dfrac{20\times0.2}{0.8} = 5\text{ N}$	A1	
	$(\pm)0.75a = 20-5$	M1	N2L
	$a = (\pm)20\text{ m s}^{-2}$	F1	
			5
(ii)	$\dfrac{20e}{0.5} = \dfrac{20(0.7-e)}{0.8}$	B1 M1 A1	$0.7-e$ equilibrium equation
	$e = 0.269$	A1	cao
			4
(iii)	if BC goes slack when speed = v		
	$\frac{1}{2}mv^2 + \dfrac{\lambda\times0.7^2}{2\times0.5} = \dfrac{\lambda\times1.2^2}{2\times0.8}$	M1	energy equation
	$\frac{1}{2}mv^2 = 8.2 > 0$ hence BC goes slack	E1	
	at furthest distance, $\dfrac{\lambda\times(\text{AB}-0.5)^2}{2\times0.5} = \dfrac{\lambda\times1.2^2}{2\times0.8}$	M1 M1 A1	energy equation EPE term in terms of a variable correct equation
	AB = 1.45 m	A1	cao
			6

13.

2(i)	$T = km^\alpha l^\beta g^\gamma \Rightarrow \text{T} = \text{M}^\alpha \text{L}^\beta (\text{LT}^{-2})^\gamma$	M1	substitute dimensions
	$\alpha = 0$	M1	equate indices and solve
	$-2\gamma = 1 \Rightarrow \gamma = -\frac{1}{2}$	A1	at least two correct indices
	$\beta+\gamma = 0 \Rightarrow \beta = \frac{1}{2}$ so $T = k\sqrt{\dfrac{l}{g}}$	A1	formula (aef)
			4
(ii)	$ml\ddot{\theta} = -mg\sin\theta$	M1 A1	N2L tangentially (no tension term) accept sign error
	$\ddot{\theta} = -\frac{g}{l}\sin\theta$	A1	
			3
(iii)	$\ddot{\theta} \approx -\frac{g}{l}\theta$ hence SHM	M1	use $\sin\theta \approx \theta$
		E1	must conclude SHM
	period $= 2\pi\sqrt{\dfrac{l}{g}}$	B1	follow their SHM equation
	i.e. as in (i) with $k = 2\pi$	B1	
			4
(iv)	$\dot{\theta} = -\theta_0\omega\sin\omega t$	M1	clear attempt at velocity (not displacement) in terms of time (must use sin or cos)
	$\lvert\dot{\theta}\rvert > \frac{1}{2}\dot{\theta}_{\max} \Leftrightarrow \lvert\sin\omega t\rvert > \frac{1}{2}$	M1	inequality for sin (or cos)
	for one half-period, $\left(0 \leqslant t \leqslant \dfrac{\pi}{\omega}\right)$ we have $\frac{1}{6}\pi < \omega t < \frac{5}{6}\pi$	M1	solve inequality
	proportion $= \dfrac{\frac{5\pi}{6\omega}-\frac{\pi}{6\omega}}{\frac{\pi}{\omega}} = \frac{2}{3}$	A1	
			4

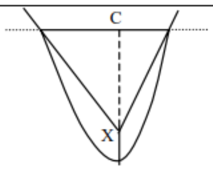
14.

3(i)	$\frac{1}{2}m\cdot 2.8^2 - mg\cdot 0.2 = \frac{1}{2}mv^2 - mg\cdot 0.2\cos\theta$	M1	attempt energy equation
	$v^2 = 2.8^2 - 0.4g + 0.4g\cos\theta$	A1	correct equation (any form)
	$v^2 = 3.92(1+\cos\theta)$	E1	
	$R - mg\cos\theta = m\dfrac{v^2}{r}$	M1 A1	N2L with $\frac{v^2}{r}$ or $r\omega^2$
	$= 19.6m(1+\cos\theta)$	M1	substitute v^2
	$R = 9.8m(2+3\cos\theta)$	E1	must use N2L in form $\sum F = ma$ or clearly justify signs
			7
(ii)	leaves when $R=0$	M1	
	$2+3\cos\theta = 0 \Rightarrow \alpha = \cos^{-1}\left(-\frac{2}{3}\right) \approx 2.3\,\text{rad} \approx 132°$	A1	
			2
(iii)	$v^2 = 3.92(1-\frac{2}{3}) \Rightarrow v = \sqrt{\dfrac{3.92}{3}} \approx 1.143$	B1	
	vertical cpt $v_y = \sqrt{\dfrac{3.92}{3}}\sin(\pi-\alpha)$	M1	
	$v_y{}^2 = 2gh \Rightarrow h = \frac{1}{27} \approx 0.037$ m	A1	
			3
(iv)	$\frac{1}{2}mv^2 - mg\cdot\frac{22}{135} = \frac{1}{2}m\cdot 2.8^2 - mg\cdot 0.2$	M1 A1	energy equation
	$v = 2.67\ \text{m s}^{-1}$	A1	cao
			3

15.

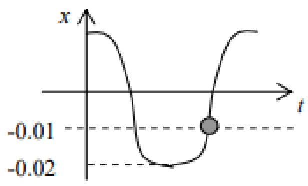
4(i)	area $= \int_0^2 (4x - x^3)\,dx = \left[2x^2 - \frac{1}{4}x^4\right]_0^2$	M1	calculate area
	$= 4$	A1	
	$4\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}\int_0^2 xy\,dx \\ \int_0^2 \frac{1}{2}y^2\,dx\end{pmatrix}$	B1 B1	$\int xy$ or $\int \frac{1}{2}y^2 dx$ seen both formulae correct
	$4\bar{x} = \int_0^2 \left(4x^2 - x^4\right)dx$	M1	integrate their expression
	$= \left[\frac{4}{3}x^3 - \frac{1}{5}x^5\right]_0^2$	A1	or multiple
	$4\bar{y} = \int_0^2 \frac{1}{2}\left(16x^2 - 8x^4 + x^6\right)dx$	M1	integrate their expression
	$= \frac{1}{2}\left[\frac{16}{3}x^3 - \frac{8}{5}x^5 + \frac{1}{7}x^7\right]_0^2$	A1	or multiple
		M1	limits
	$\bar{x} = \frac{16}{15}$	E1	
	$\bar{y} = \frac{128}{105}$	E1	
			11

(ii)



Forces concurrent	B1	(or correct moments equation)
$CX = \frac{14}{15}\tan 60°$	M1	(or attempt two other equations)
$\frac{16}{15}\tan\alpha = CX = \frac{14}{15}\sqrt{3}$	M1	(or eliminate tensions)
$\Rightarrow \alpha = 56.6°$	A1	
		4

16.

(a) (i)	$[a] = [x] = \mathrm{L}$ $[v] = \mathrm{L\,T^{-1}}$	B1		
	$\mathrm{L^2\,T^{-2}} = [\omega^2]\,\mathrm{L^2}$	M1	Equating	
	so $[\omega] = \mathrm{T^{-1}}$	E1	[Award max of 2 if any units used instead of dimensions]	3
(ii)	angular speed frequency	B1	cao. Accept ang vel. Accept examples e.g. pulse rate	1
(b) (i)	$F = ky \Rightarrow 0.015\times 9.8 = 0.098k$	M1	$F = mg$ (0.147) and $y = 0.098$ in $F = ky$. Give for $k = \dfrac{0.015g}{0.098}$ seen or implied.	
	$\Rightarrow k = 1.5$	E1	Fully explained. Accept $F = ky$ not explained.	2
(ii)	N2L $\downarrow$ $0.015\times 9.8 - (0.098 + x)\times 1.5 = 0.015\ddot{x}$	M1 B1 A1	N2L applied including attempts at weight and upthrust Upthrust term. Allow $\pm(0.098 + x)\times 1.5$. All correct including signs. Accept $x \uparrow$ and using y	
	$\Rightarrow \ddot{x} + 100x = 0$	E1	Clearly shown with given x.	4
(iii)	$v^2 = 100(0.02^2 - 0.01^2)$	M1	Use of this result or differentiation etc	
	$\Rightarrow v = 0.1732...$ so $0.173\ \mathrm{m\,s^{-1}}$ $\left(\sqrt{0.03}\right)$	A1	[Using a W-E approach M1 for GPE, KE and WD against F term attempted. A1] [If time found first,: M1 for $\dot{x}$ (**their** time) used. A1]	
	$x = 0.02\cos 10t$	B1	Or equivalent	
	we need $-0.01 = 0.02\cos 10t$ $\Rightarrow \cos 10t = -0.5$	M1	Equating in **their** expression for x. Allow sign error.	
	$\Rightarrow 10t = \dfrac{4\pi}{3}$ so $t = \dfrac{2\pi}{15}$ (= 0.418879… so 0.419 s (3 s. f.)	A1	cao	
	 		[If graphical method used: B1 shape; B1 x = -0.01 line; B1 cao]	5
			total	15

17.

(i)	No tangential acceleration	E1		1
(ii)	$R = mr\omega^2 = 0.3 \times 0.4 \times 100$ $= 12$ so 12 N	M1 A1	N2L radially. (Award for v^2/r with $v = 10$)	2
(iii)	$\uparrow \quad F = 0.3g \ (2.94)$ $F \le \mu R$ so $\mu \ge \frac{g}{40}$ so least value is $\frac{g}{40}$ (0.245)	B1 M1 A1	Accept inequality Accept =. Only vertical F considered. Use **their** R from (ii). Accept inequality inc strict. FT value of R from (ii).	3
(iv)	$R = mr\omega^2 = 0.12(10+5t)^2$ $= 3(2+t)^2$	M1 E1	N2L radially and substitute for ω. (Not v^2/r with $v = (10 + 5t)$) Clearly shown	2
(v)	$F = mr\dot{\omega} = 0.3 \times 2 = 0.6$ $F \le \mu R \Rightarrow \mu \ge \frac{0.6}{3(2+t)^2} = \frac{1}{5(2+t)^2}$ We need the greatest value μ can take which is when $t = 0$. This gives $\mu \ge 0.05$.	B1 M1 E1 E1 E1	N2L transverse direction Condone = and $\le$. R must be correct. F must be attempted from consideration of transverse motion. Clear explanation required Accept =. Dependent on correct reason given.	5
			total	13