Mech Minor – Past Paper Pack 2

Questions made from MEI papers between 2003 and 2005

All Mech Minor topics included except for couples.

1.

My cat Jeoffry has a mass of 4 kg and is sitting on rough ground near a sledge of mass 8 kg. The sledge is on smooth, horizontal ice.

Initially, the sledge is at rest and Jeoffry jumps and lands on it when moving with a horizontal speed of 2.25 m s^{-1} parallel to the runners of the sledge. On landing, Jeoffry grips the sledge with his claws so that he does not move relative to the sledge in the subsequent motion.

(i) Show that, after Jeoffry lands on it, the sledge moves off with a speed of 0.75 m s^{-1}. [2]

With the sledge and Jeoffry moving at 0.75 m s^{-1}, the sledge collides *directly* with a stationary stone of mass 3 kg. The stone may move freely on the ice. The coefficient of restitution in the collision is $\frac{4}{15}$.

(ii) Calculate the speed of the sledge and Jeoffry after the collision. [6]

In a new situation, Jeoffry is initially at rest on the sledge when it is stationary. He then walks from the back to the front of the sledge.

(iii) Giving a brief reason for your answer, describe the motion of the common centre of mass of Jeoffry and the sledge during his walk. [2]

Jeoffry is sitting on the sledge when it is stationary. He now jumps off. After he has left the sledge, his horizontal speed relative to the sledge is 3 m s^{-1}.

(iv) With what speed is the sledge travelling after Jeoffry leaves it? [4]

[Total 14]

2.

A roller-coaster car at a fairground has a mass of 380 kg.

At the start of a ride, the car is pulled up a slope at a steady speed of $v\,\mathrm{m\,s^{-1}}$ by a force of 2160 N. The power of the force is 3780 W.

(i) Calculate the value of v. [2]

The pulling force is removed. The car now moves under gravity and against resistances to its motion.

At a point P, the car is 20 m above horizontal ground and is travelling at $2\,\mathrm{m\,s^{-1}}$. It then travels 100 m along the track in 20 seconds to a point Q that is 5 m above the ground, as shown in Fig. 2. At Q it is travelling at $10\,\mathrm{m\,s^{-1}}$.

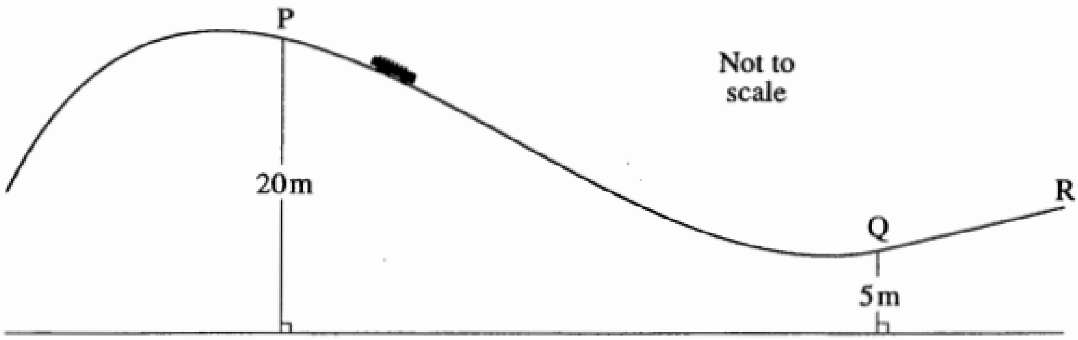


Fig. 2

(ii) For the motion from P to Q, calculate

(*A*) the change in the kinetic energy of the car,

(*B*) the change in the gravitational potential energy of the car,

in each case stating whether the change is a gain or loss. [5]

During the motion from P to Q, the average braking force over the distance is 150 N.

(iii) Show that the total work done against the resistances to motion *other than braking* is 22 620 J. [3]

At the point Q, the car goes up a uniform slope at 20° to the horizontal and comes instantaneously to rest at the point R. The average resistances to motion and the average braking force from Q to R are the same as in the motion from P to Q.

(iv) Calculate the vertical distance of R above Q. [6]

[Total 16]

3.

An injured climber is tied to a stretcher AB of length 2.5 m. The total mass of the climber and the stretcher is 90 kg.

In each part of this question you should make the following modelling assumptions:

- the centre of mass, G, of the stretcher with climber is a distance 1.875 m from the end A of the stretcher, as shown in Fig. 3.1,
- all the forces acting on the system are in the same vertical plane.

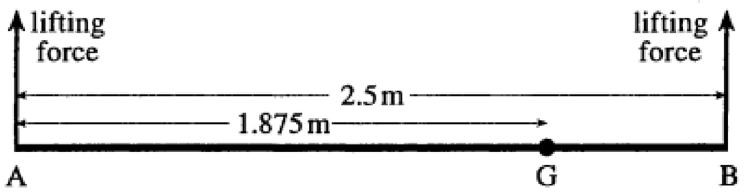


Fig. 3.1

In one situation, the lifting forces are each vertically upwards at the ends A and B of the stretcher, and the stretcher is held in horizontal equilibrium, as shown in Fig. 3.1.

(i) Calculate the values of the lifting forces. [5]

In another situation, the end A of the stretcher is resting on rough horizontal ground. The stretcher is held in equilibrium at 15° to the horizontal by a force at B that is at 65° to the horizontal, as shown in Fig. 3.2.

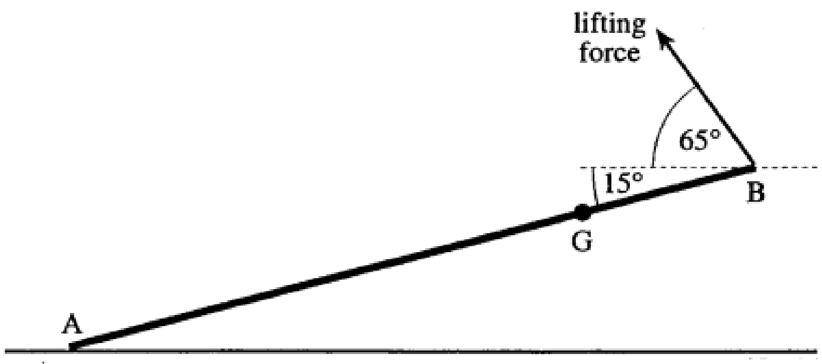


Fig. 3.2

(ii) Show that the lifting force is 649 N, correct to three significant figures. [6]

(iii) The stretcher is on the point of sliding. Calculate the coefficient of friction between the end of the stretcher and the ground. [5]

[Total 16]

4.

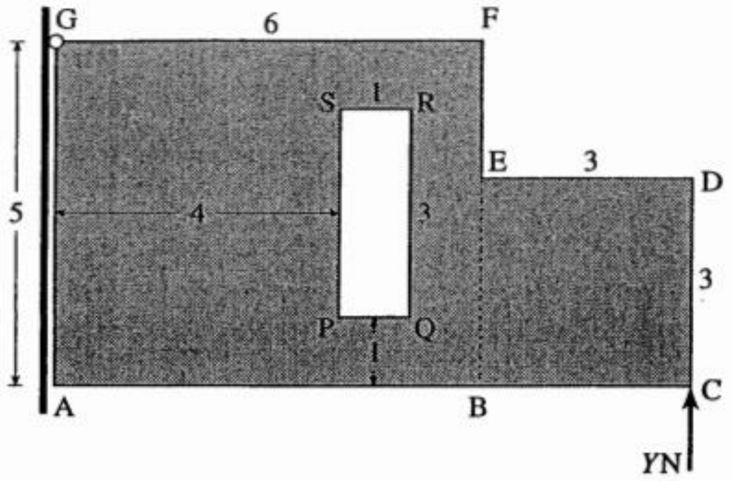


Fig. 4.1

Fig. 4.1 shows a pre-fabricated section of a building ABCDEFG. This section, which is shaded, is a lamina of uniform material made up of a rectangle ABFG (with a rectangle PQRS cut out) and a square BCDE. The dimensions of the section are shown in Fig. 4.1. The lengths are in metres.

The section is freely attached to a wall at G and is held in equilibrium by a vertical force Y N at C. The weight of the shaded section is 3150 N. AC is horizontal.

(i) Calculate the horizontal distance from the line AG of the centre of mass of the shaded section. [4]

(ii) Show that $Y = 1400$. [3]

5.

Newton's Law of Gravitation states that the gravitational force, F, between two bodies of masses m_1 and m_2 a distance r apart is given by

$$F = \frac{Gm_1m_2}{r^2},$$

where G is the universal gravitational constant.

(i) Use this equation to show that the dimensions of G are $M^{-1}L^3T^{-2}$. [3]

A ray of light passing close to a star is bent by the gravitational field of the star. It is suggested that the bending angle, θ, can be expressed as

$$\theta = kG^{\alpha}m^{\beta}r^{\gamma}c^{\delta},$$

where m is the mass of the star, r is the distance between the ray of light and the star, c is the speed of light, G is the universal gravitational constant and k is a dimensionless constant.

(ii) Show that an angle in radians is dimensionless. [1]

(iii) Assuming that the above relationship is correct,

(*A*) explain why the values of α, β, γ and δ cannot be determined by dimensional analysis, [2]

(*B*) use dimensional analysis to find β, γ and δ in terms of α and hence show that

$$\theta = k\left(\frac{Gm}{rc^2}\right)^{\alpha}.$$ [6]

Two observations are made of light passing a particular star.

Observation	Distance, r	Angle, θ
1	2×10^9 m	3×10^{-5} rad
2	1.5×10^9 m	4×10^{-5} rad

(iv) Use these figures to determine α. [2]

(v) Given also that the mass of the star is 2.015×10^{31} kg, $G = 6.7 \times 10^{-11}\ m^3\,s^{-2}\,kg^{-1}$ and $c = 3.0 \times 10^8\ m\,s^{-1}$, calculate the value of k correct to 3 significant figures. [1]

6.

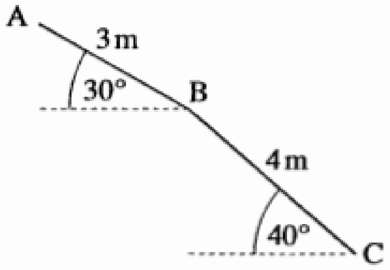


Fig. 2

A small tile of mass 0.5 kg slides down a roof. It slides 3 m from A to B at 30° to the horizontal and, without losing contact, slides 4 m from B to C at 40° to the horizontal. The coefficient of friction, μ, between the tile and the roof is the same for both parts of the motion.

(i) Calculate the gravitational potential energy lost by the tile in travelling from A to C. [3]

(ii) Write down expressions in terms of μ for the work done against friction by the tile as it travels from A to B and from B to C. [3]

The tile has the same speed at C as at A.

(iii) Show that the value of μ is about 0.72. [3]

The speed of the tile at A is 3.5 m s^{-1}.

(iv) Calculate the speed of the tile at B. [5]

[Total 14]

7.

In this question, the units of the axes shown in the diagrams are centimetres and all coordinates ar referred to these axes.

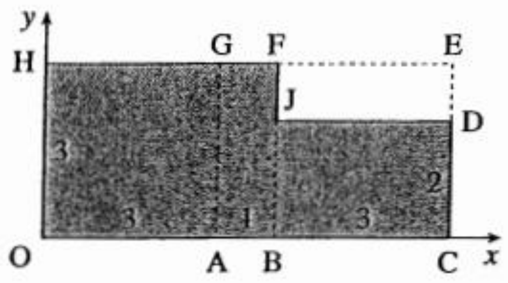


Fig. 3.1

A uniform lamina OCDJFH is in the shape of the rectangle OCEH with the rectangle DEFJ removed, as shown in Fig. 3.1.

(i) Calculate the coordinates of the centre of mass of OCDJFH. [5]

The lamina is now folded along AG and BJ to form the shape shown in Fig. 3.2. Angles OAB and ABC are 90°.

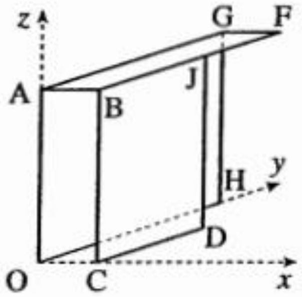


Fig. 3.2

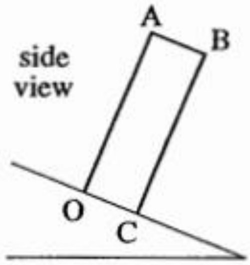


Fig. 3.3

(ii) Calculate the coordinates of the centre of mass of the folded lamina. [5]

The folded lamina is placed on an inclined plane with OC along a line of greatest slope, as shown in Fig. 3.3. The plane is tilted slowly until the lamina is about to turn about the edge CD. At this stage, the lamina is also about to slip down the plane and OC is inclined at an angle λ to the horizontal. The coefficient of friction between the folded lamina and the plane is μ.

(iii) Show that $\tan\lambda = \mu$.

Draw a diagram showing the line of action of the weight of the lamina.

Calculate the value of μ. [5]

[Total 15]

8.

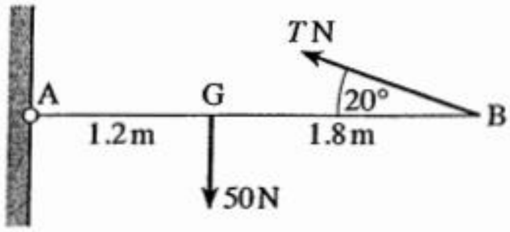


Fig. 4.1

A heavy beam AB of length 3 m is freely hinged at A and its weight of 50 N acts through its centre of mass G, where AG is 1.2 m. It is held horizontally in equilibrium by a force of magnitude T N acting at 20° to the horizontal, as shown in Fig. 4.1.

Calculate the value of T. [3]

9.

In this question about a spacecraft, you may assume that the resistance to motion and the gravitational attraction are negligible, that all the motion is in the same straight line and that all the collisions are direct. The initial direction of motion is taken as positive.

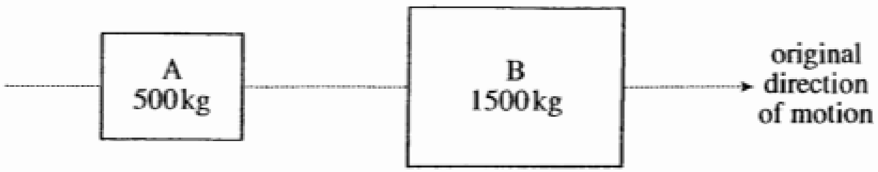


Fig. 1

A spacecraft of mass 2000 kg travelling at 100 m s^{-1} separates into parts A and B. As shown in Fig. 1, A has a mass of 500 kg and B a mass of 1500 kg.

After separation the speed of B is 150 m s^{-1} in the original direction of motion.

(i) Calculate the impulse acting on B in the separation. [2]

(ii) Calculate the velocity of A after the separation. [3]

(iii) Given that the separation takes place over 75 seconds, calculate the average separating force. [2]

B meets and attaches to a satellite C of mass 500 kg travelling in the same direction as B with speed 130 m s^{-1}. The combined B and C is referred to as D.

(iv) Show that the velocity of D is 145 m s^{-1} in the original direction of motion. [3]

An impulse now acts on A so that it approaches D at a relative speed of 10 m s^{-1}. There is then a perfectly elastic collision between A and D (that is, a collision with $e = 1$).

(v) Calculate the velocity of A and the velocity of D after the collision. [6]

[Total 16]

10.

(a) A grandfather clock is driven for two days by a 4 kg mass of brass falling through a vertical distance of 80 cm.

What power is required to drive the clock? [3]

(b)

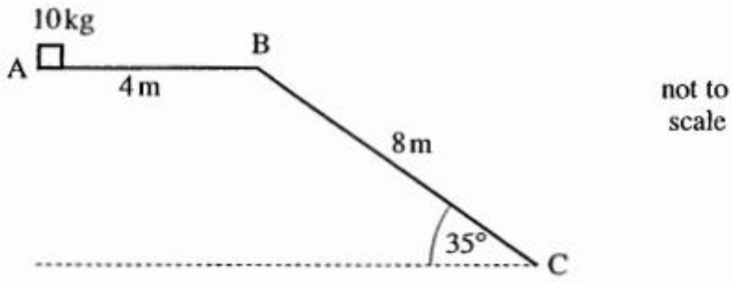


Fig. 2

Fig. 2 shows a small block of mass 10 kg set in motion along a path ABC. The block first travels a distance of 4 m horizontally from A to B against a constant resistance of 40 N. Its speed at A is $v \text{ m s}^{-1}$.

(i) Calculate the least possible value of v. [3]

Without losing contact, the block now slides 8 m down a ramp from B to C. The ramp is inclined at 35° to the horizontal.

(ii) Calculate the gravitational potential energy lost by the block as it slides from A to C. [2]

In a particular case, $v = 6$. From B to C the block moves against a constant resistance of 50 N.

(iii) Calculate the speed of the block at C. [6]

[Total 14]

11.

(a) A cuboid of weight W N is in equilibrium on a horizontal table. The cuboid is a m long and the centre of mass is b m from end P. A vertical force, T N, acts on it at end Q. The reaction force, R N, of the table on the cuboid acts x m from end P, as shown in Fig. 3.

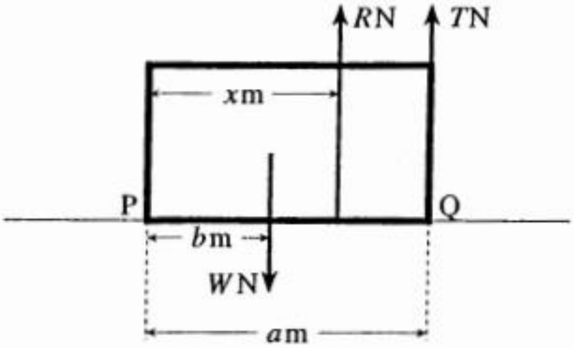


Fig. 3

(i) By resolving and taking moments, write down two equations for the equilibrium of the cuboid.

Hence show that $x = \dfrac{Wb - Ta}{W - T}$. [5]

(ii) Find an expression for R in terms of W, a and b when the cuboid is on the point of turning about the edge through P. [2]

(b) A uniform ladder rests against a smooth vertical wall and on rough horizontal ground. It is inclined at 72° to the horizontal when it is about to slip.

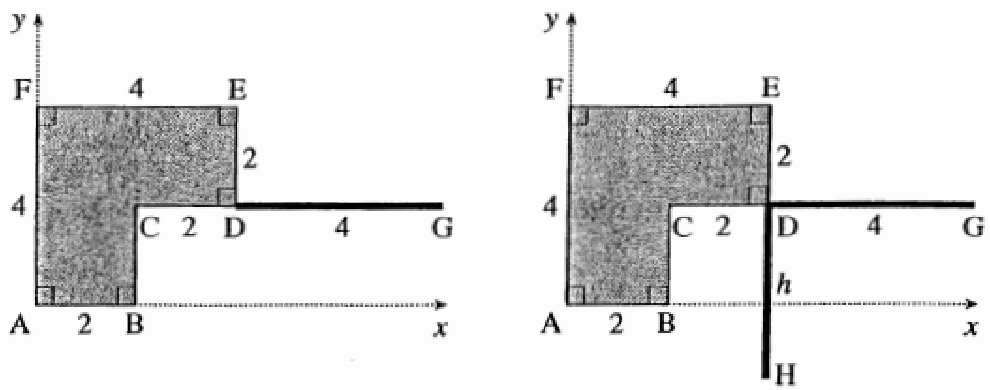
Draw a diagram showing the forces acting on the ladder.

Calculate the coefficient of friction between the ladder and the ground. [8]

[Total 15]

12.

Fig. 4.1 shows a uniform lamina ABCDEF. The units are metres. The lamina has a weight of 1 N per unit area. The rod DG is 4 m long and rigidly attached to the lamina so that CDG is a straight line. The weight of the rod is 1 N per unit length.



not to scale

Fig. 4.1 **Fig. 4.2**

(i) Calculate the coordinates of the centre of mass of the lamina and rod, referred to the axes shown in Fig. 4.1. [5]

A further rod, DH, of length h m is made of the same material as DG. This rod is rigidly attached to the lamina so that EDH is a straight line, as shown in Fig. 4.2.

(ii) Referred to the axes shown in Fig. 4.2, show that the coordinates of the centre of mass of the lamina and two rods are

$$\left(\frac{44+4h}{16+h}, \frac{72+h(4-h)}{2(16+h)}\right).$$ [6]

The composite figure shown in Fig. 4.2 is freely suspended from M, the mid-point of AF, and hangs in equilibrium with B vertically below M.

(iii) Show that, referred to the axes shown in Fig. 4.2, its centre of mass must lie on the line $y = 2 - x$. Calculate the value of h. [4]

[Total 15]

13.

(i) State the dimensions of frequency and density. Show that the dimensions of pressure are $\mathrm{M\,L^{-1}\,T^{-2}}$. [4]

The frequency, f, of the note emitted by an organ pipe of length a when the air pressure is p and the air density is d is believed to obey a law of the form

$$f = k a^{\alpha} p^{\beta} d^{\gamma} \,,$$

where k is a dimensionless constant.

(ii) Determine the values of α, β and γ. [6]

A particular organ pipe is tuned to a frequency of 440 Hz. Subsequently the air density falls by 2% and the air pressure rises by 0.5%. Any change in the length of the pipe is negligible.

(iii) Calculate the new frequency of the organ pipe. [4]

[Total 14]

14.

A uniform beam AB of length 3 m and weight 80 N is freely hinged at A.

Initially, the beam is held horizontally in equilibrium by a small, smooth peg at C where the distance AC is 2.5 m, as shown in Fig. 1.1.

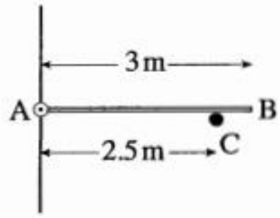


Fig. 1.1

(i) Calculate the force on the beam from the peg at C. [3]

The peg is now moved so that the beam is in equilibrium with AB at 60° to the vertical, as shown in Fig. 1.2. AC is still 2.5 m.

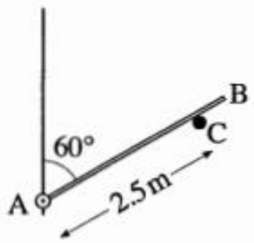


Fig. 1.2

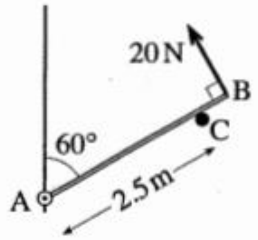


Fig. 1.3

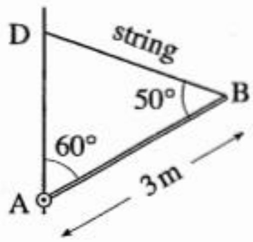


Fig. 1.4

(ii) Calculate the new force on the beam due to the peg. [4]

A light string is now attached to the beam at B. The string is perpendicular to the beam. The beam is in equilibrium with a tension of 20 N in the string, as shown in Fig. 1.3.

(iii) Calculate the new force on the beam due to the peg. [2]

The peg is now removed and the string attached to a point D vertically above A so that angle ABD is 50°, as shown in Fig. 1.4.

(iv) Calculate the new tension in the string. Calculate also the vertical component of the force acting on the hinge at A. [5]

[Total 14]

15.

A small block of mass 25 kg is on a rough slope inclined at $\alpha°$ to the horizontal. The block is held in equilibrium by a force of magnitude P N applied parallel to the slope and up the slope, as shown in Fig. 2.

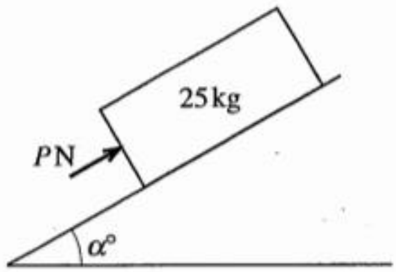


Fig. 2

When $P = 259$, the block is about to slip **up** the slope. When $P = 35$, the block is about to slip **down** the slope. In each case, the magnitude of the frictional force acting on the block is F N.

(i) Draw separate force diagrams for these two cases, showing all the forces and making clear the direction in which the frictional force acts. [2]

(ii) Calculate the value of α and show that $F = 112$. [5]

(iii) Calculate the coefficient of friction between the block and the slope. [3]

The force of magnitude P N is removed and the block slides from rest down the slope. The slope is not uniformly smooth but the frictional force averages 112 N over the distance slid.

The speed of the block is to be calculated after it has slid 3 m.

(iv) (*A*) Explain briefly why a method using Newton's second law with the constant acceleration formulae is not appropriate.

(*B*) Use an energy method to calculate the speed. [6]

[Total 16]

16.

A lamina ACDEFI is made from uniform material. ACGI and DEFG are both rectangular. The dimensions in centimetres are shown in Fig. 3.1.

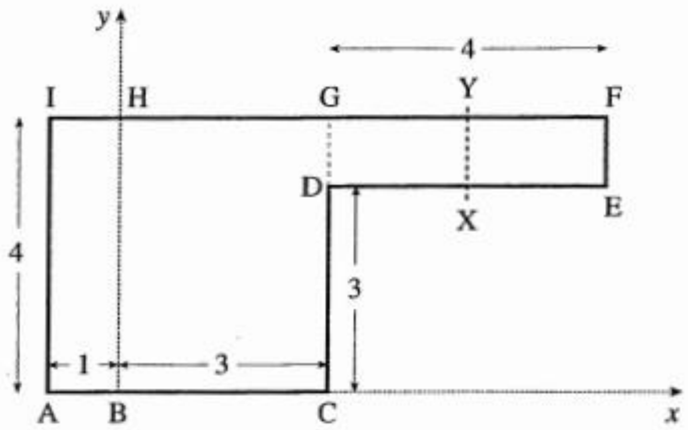


Fig. 3.1

(i) Calculate the coordinates of the centre of mass of the lamina ACDEFI, with respect to the axes shown in Fig. 3.1. [5]

ABHI is now folded along BH so that it is perpendicular to the plane BCGH. XEFY is folded along XY so that it is also perpendicular to the plane BCGH but on the other side of it. This situation is shown in Fig. 3.2. XY is parallel to EF and the distance EX is such that the centre of mass of the folded lamina remains in the plane BCGH.

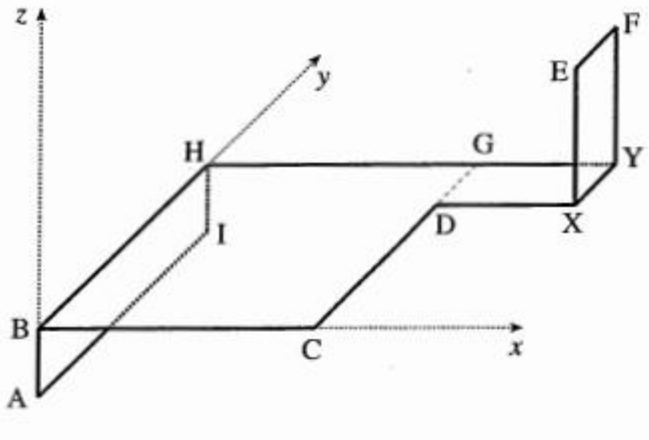


Fig. 3.2

(ii) Verify that the distance EX is 2 cm. [4]

(iii) Calculate the x- and y-coordinates of the centre of mass of the folded lamina with respect to the axes shown in Fig. 3.2. [4]

(iv) The folded lamina is freely suspended from H and hangs in equilibrium with HBCDXYG in a vertical plane. Calculate the angle between HB and the vertical. [3]

[Total 16]

17.

Two circular discs of equal radius slide on a smooth, horizontal surface. Disc A has mass 2 kg and disc B has mass m kg. All impacts are direct and motion is along the line of centres, which is perpendicular to a wall. The situation is shown in Fig. 4.

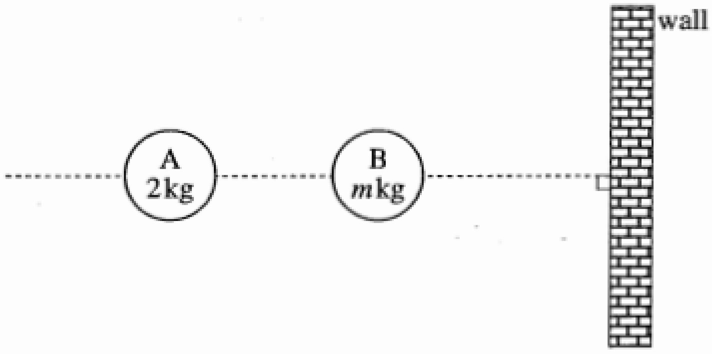


Fig. 4

A force of 4 N acts on A for 5 seconds along the line of centres in the direction AB. Disc A is initially at rest and does not reach disc B in the 5 seconds.

(i) Show that A achieves a speed of $10\,\mathrm{m\,s^{-1}}$ in the direction AB. [1]

Suppose that $m = 5$, B is initially at rest and the coefficient of restitution in the impact between A and B is 0.54.

(ii) Show that, after the impact, A has a speed of $1\,\mathrm{m\,s^{-1}}$ in the direction BA and B has a speed of $4.4\,\mathrm{m\,s^{-1}}$ in the direction AB. [5]

Disc B rebounds from the wall with its speed halved.

(iii) Calculate the impulse of the wall on B. [2]

Consider now the more general case of the collision between A and B where B is initially at rest. In this general case, B has mass m kg and the coefficient of restitution in the impact between A and B is e. Disc A travels at $10\,\mathrm{m\,s^{-1}}$ in the direction AB and collides with B. After the collision, A has speed $1\,\mathrm{m\,s^{-1}}$ in the direction BA.

(iv) (*A*) Show that $e = \dfrac{22 + m}{10m}$.

(*B*) The speed of B is halved in its impact with the wall. Disc B collides again with A after the impact with the wall. Show that $m < 11$ and find the range of possible values of e. [6]

[Total 14]