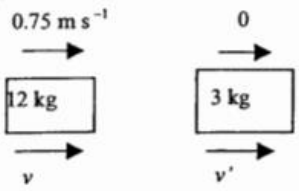
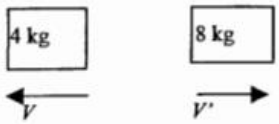
Mech Minor – Mark Scheme

1.

(i)	$4 \times 2.25 + 0 = 12v$	M1	Use of PCLM	
	$v = 0.75$ so $0.75\ \text{m s}^{-1}$	E1		2
(ii)	before after  			
	PCLM →	B1	Award if final answer correct	
	$9 = 12v + 3v'$ so $3 = 4v + v'$	M1 A1	Use of PCLM. Allow coalescence. Any form	
	NEL →			
	$\frac{v'-v}{0-0.75} = -\frac{4}{15}$ so $v'-v = 0.2$	M1 A1	Use of NEL Any form	
	Solving $v = 0.56$ so $0.56\ \text{m s}^{-1}$ (in original direction)	A1	Cao. Direction not required.	6
(iii)	Does not move No horizontal external force acts on the cat-sledge system	B1 B1	No external force or momentum is zero or…	2
(iv)	 			
	PCLM	M1	Dealing with relative motion	
	$4V = 8V'$	A1		
	$V + V' = 3$	B1		
	Solving $V' = 1$ so $1\ \text{m s}^{-1}$	A1		4
		Tot 14		

2.

(i)	$P = Fv$ so $3780 = 2160v$ and $v = 1.75$	M1 A1	Use of $P = Fv$	2
(ii) (A)	$\frac{1}{2}\times 380\times 10^2 - \frac{1}{2}\times 380\times 2^2$ $= 18240$ J (Gain)	M1 A1	$0.5mv^2$ used Gain not required	
(B)	$380\times 9.8\times 15$ $= 55860$ J (Loss)	M1 A1	mgh used Loss not required	
		B1	Gain/loss both correct	5
(iii)	$55860 - 18240 - 150\times 100$	M1 B1	All terms present (accept wrong signs) 15000. Accept WW.	
	$= 22620$ J	E1		3
(iv)	**Either** $0.5\times 380\times 10^2$ $= 380\times 9.8\times h + \left(150 + \frac{22620}{100}\right)\times \frac{h}{\sin 20}$	M1 B1 B1 A1 A1	Use of WE inc GPE, KE and one resistance term 226.2 (FT their value for WD) h/sin20 Total WD against resistances correct. All terms correct but allow sign errors.	
	$h = 3.9386\ldots$ so 3.94 m (3 s. f.)	A1	cao	6
	or			
	$-380\times 9.8\sin 20 - 150 - 226.2 = 380a$	M1 B1 A1	Use of N2L with weight and at least one resistance term For 226.2 N All correct. Accept LHS terms all + ve	
	$a = -4.34179\ldots$	A1		
	$0^2 = 100 + 2\times -4.34179\ldots \times s$	F1	Accept lack of consistency with signs. FT wrong a.	
	$h = s\sin 20 = 3.9386\ldots$ so 3.94 m (3 s. f.)	A1	cao	
		Tot 16		

3.

(i)	a.c. moments about A			
	$T_B \times 2.5 - 90 \times 9.8 \times 1.875 = 0$	M1 A1	Attempt at moments	
	$T_B = 661.5$ so 661.5 N	A1		
	Resolve vert	M1	Resolve or moments about an appropriate point	
	$T_A + T_B = 90 \times 9.8$			
	$T_A = 220.5$ so 220.5 N	A1	[If g omitted; SC award M1 A0 A0 M1 F1 maximum]	5
(ii)	a.c. moments about A $T \times 2.5\cos 10$	M1 M1 A1	Moments about A Attempt at resolution. Allow sin ↔ cos LHS; correct component of force or displacement	
	$= 90g \times 1.875\cos 15$	A1 A1	RHS; correct component of weight or displacement All correct	
	$T = 648.816..$ so 649 N (3 s. f.)	E1	[If g omitted; SC award M1 M1 A1 F1 A0 E0 maximum]	6
(iii)	$R = 90g - T\cos 25$	M1	Attempt at R including a cpt of T	
	$= 293.9721...$	A1		
	$F = T\cos 65 = 274.201...$	B1	Allow for $T\cos 65$ seen	
	$\mu = \dfrac{274.201...}{293.9721...} = 0.9327..$	M1	Use of $F = \mu R$	
	so 0.933 (3 s. f.)	A1	cao	5
		Tot 16		

4.

(i)	Let distance be x $(30 - 3 + 9)\bar{x} = 30 \times 3 - 3 \times 4.5 + 9 \times 7.5$	M1 A1 A1	Correct method for c.m. Masses correct At least 2 x values correct	
	$\bar{x} = 4$	A1	[Award 4/4 for $\bar{x} = 4$ seen WW]	4
(ii)	a.c. moments about G	M1	Attempt at moments	
	$9Y - 4 \times 3150 = 0$	A1	FT	
	$Y = 1400$ so 1400 N	E1	Clearly shown	3

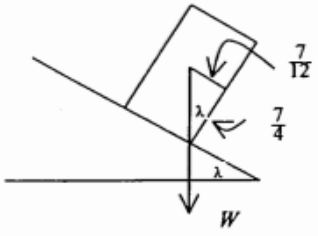
5.

2(i)	$[G] = \dfrac{MLT^{-2} \cdot L^2}{M \cdot M}$ $\Rightarrow [G] = M^{-1}L^3T^{-2}$	M1 A1 E1	substitute dimensions correct dimensions	3
(ii)	e.g. $s = r\theta \Rightarrow [\theta] = \frac{L}{L} = 1$	E1	any valid reason	1
(iii)	(*A*) dimensional analysis gives only 3 equations but 4 unknowns so cannot solve	B1 B1	for indication that method provides less information than needed for full explanation	2
	(*B*) $1 = (M^{-1}L^3T^{-2})^{\alpha} M^{\beta} L^{\gamma} (LT^{-1})^{\delta}$ M: $-\alpha + \beta = 0$ L: $3\alpha + \gamma + \delta = 0$ T: $-2\alpha - \delta = 0$ $\beta = \alpha, \delta = -2\alpha, \gamma = -\alpha$ $\theta = kG^{\alpha} m^{\alpha} r^{-\alpha} c^{-2\alpha} = k\left(\frac{Gm}{rc^2}\right)^{\alpha}$	M1 M1 M1 A1 A1 E1	substitute dimensions one equation good attempt at all three equations one correct all correct. Evidence of working is needed as can be deduced from given answer. Use of given answer (verification) earns at most 3 marks: M1 M1 M0 A1 A0 E0. must be shown, not just stated	6
(iv)	$\frac{3 \cdot 10^{-5}}{4 \cdot 10^{-5}} = \left(\frac{1.5 \cdot 10^{9}}{2 \cdot 10^{9}}\right)^{\alpha}$ $0.75 = 0.75^{\alpha} \Rightarrow \alpha = 1$	M1 A1	eliminate constants must have evidence of working	2
(v)	$k = \frac{3\times10^{-5}\times2\times10^{9}\times(3.0\times10^{8})^2}{6.7\times10^{-11}\times2.15\times10^{31}} = 4.00$	B1	cao (must be to 3sf)	1

6.

(i)	$(3\sin 30 + 4\sin 40)\times 0.5\times 9.8$	M1 A1	Use of mgh Either GPE term correct	
	19.94867... so 19.9 J (3 s. f.)	A1		3
(ii)	$3\times 0.5g\cos 30\mu$ (12.73057...μ)	M1 M1	Use of $F = \mu R$. Accept g not used. Use of WD = Fd	
	$4\times 0.5g\cos 40\mu$ (15.01447...μ)	A1	Both. Any form.	3
(iii)	$(3\times 0.5g\cos 30 + 4\times 0.5g\cos 40)\mu$ $= 19.94867...$	M1 A1	Use of work-energy equation	
	so $\mu = 0.71899...$, so 0.72 (2 s. f.) **or**	E1	Accept any reasonable accuracy	
	N2L to find a and *uvast* Eliminate velocities so $\mu = 0.71899...$, so 0.72 (2 s. f.)	M1 A1 E1	Used on both sections. Accept use of 3.5 for v_A Statement correct in any form Accept any reasonable accuracy	3
(iv)	$0.5\times 0.5\times 3.5^2 + 0.5\times 9.8\times 3\sin 30$ $-3\times 0.5\times 9.8\cos 30\mu = 0.5\times 0.5v^2$	M1	work-energy equation with at least two of KE, GPE, WD against friction	
		B1 B1 A1	Use of $0.5mv^2$ for KE and Δ KE GPE term correct All correct	
	$v = 2.2443...$ so 2.24 m s^{-1} (3 s. f.) **or**	A1	cao. Accept 2.23.	
	N2L on AB Use of appropriate *uvast*	M1 A1 M1 F1 A1	All forces present Expression or $a = 1.21..$ or $-1.21...$ Dep on 1st M1. Signs consistent FT **their** a cao Accept 2.23.	5
			total	14

7.

(i)	$18\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=21\begin{pmatrix}3.5\\ 1.5\end{pmatrix}-3\begin{pmatrix}5.5\\ 2.5\end{pmatrix}=\begin{pmatrix}57\\ 24\end{pmatrix}$ $\bar{x}=\frac{19}{6}$, $\bar{y}=\frac{4}{3}$	M1 B1 B1 A1 A1	Correct method Masses correct At least one c.m. correct (if x, y done separately, at least two c. m. cpts correct)	5
(ii)	$\bar{y}=\frac{4}{3}$. No change. $18\begin{pmatrix}\bar{x}\\ \bar{y}\\ \bar{z}\end{pmatrix}=9\begin{pmatrix}0\\ 1.5\\ 1.5\end{pmatrix}+3\begin{pmatrix}0.5\\ 1.5\\ 3\end{pmatrix}+6\begin{pmatrix}1\\ 1\\ 1.5\end{pmatrix}$ $\bar{x}=\frac{5}{12}$, $\bar{z}=\frac{7}{4}$	B1 M1 B1 A1 A1	Accept WW. Or work out below. Correct method dealing with fold At least one c.m. correct (if x, y, z done separately, at least two c. m. cpts correct)	5
(iii)	Since about to slip, $F=\mu R$ $F=mg\sin\lambda$ and $R=mg\cos\lambda$ Hence $\tan\lambda=\mu$ Line of action of weight acts through CD  $\mu=\tan\lambda=\frac{7}{12}\div\frac{7}{4}=\frac{1}{3}$	M1 A1 B1 M1 A1	Use of $F=\mu R$ and attempts to find F and R Clearly shown Identify both relevant lengths and correct angle (accept the use of $\frac{5}{12}$ in place of $\frac{7}{12}$ but not $\frac{4}{3}$ instead of $\frac{7}{4}$) cao	5
			total	15

8.

$\hat{A}$ $1.2\times50=3T\sin20$ T = 58.4760… so 58.5 N (3 s. f.)	M1 B1 A1	Moments about A, or equivalent $3T\sin20$	3

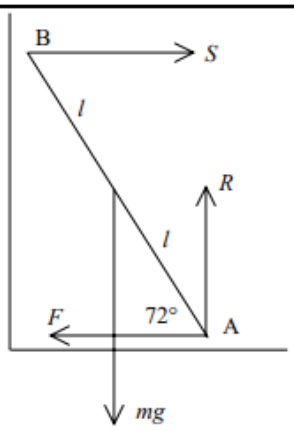
9.

(i)	→ $1500(150-100) = 75000$ N s in original direction	 M1 A1	 $m\mathbf{v} - m\mathbf{u}$ Accept +ve answer without direction	2
(ii)	**either** → $500\times100 - 75000 = 500v$ $v = -50$ so 50 m s^{-1} in opposite direction to original motion **or** → $2000\times100 = 500v + 1500\times150$ $v = -50$ so 50 m s^{-1} in opposite direction to original motion	 M1 B1 A1 M1 A1 A1	 Impulse-momentum, both terms Use of **their** –75000 or equivalent Accept –ve answer without direction Use of PCLM	3
(iii)	$F\times75 = 75000$ $F = 1000$ so 1000 N	M1 A1	Use of $\mathbf{I} = t\mathbf{F}$ FT **their** 75 000	2
(iv)	→ $1500\times150 + 500\times130 = 2000V$ $V = 145$ so 145 m s^{-1}	 M1 B1 E1	 PCLM Deal with coalescence	3
(v)	→ $u_A = 155$ $500\times155 + 2000\times145 = 500v_A + 2000v_D$ so $735 = v_A + 4v_D$ $v_D - v_A = 10$ Solving $v_D = 149$ so 149 m s^{-1} $v_A = 139$ so 139 m s^{-1}	 B1 M1 A1 B1 A1 A1	 PCLM Any form Any form; condone wrong sign. cao cao	6
			total	16

10.

(a)	$\frac{4\times9.8\times0.8}{2\times24\times3600}=181\,\mu$ W (3 s. f.)	M1 M1 A1	Use of *mgh* for GPE Use of WD/time	3
(b) (i)	**either** Must not stop before the start of the ramp so $0.5\times10\times v^2\geq4\times40$ least v is $4\sqrt{2}$ m s^{-1} or 5.66 m s^{-1} (3 s. f.) **or** Use of N2L to give $a=-4$ $v_B^2=v^2+2\times-4\times4$ $v_B\geq0$ least v is $4\sqrt{2}$ m s^{-1} or 5.66 m s^{-1} (3 s. f.)	M1 A1 A1 B1 M1 A1	Expression attempted involving WD and KE Correct. Accept =. Accept ≥ Allow use of **their** a in appropriate *uvast* Accept ≥	3
(ii)	$10\times9.8\times8\sin35$ $=449.6839\ldots$ so 450 J (3 s. f.)	M1 A1	Use of cpt in GPE change cao	2
(iii)	**either** $0.5\times10\times36+449.6839...-4\times40-8\times50$ $=0.5\times10V^2$ $V=3.73319\ldots$ so 3.73 m s^{-1} (3 s. f.) **or** $v_B=2$ N2L on BC $10g\sin35-50=10a$ $a=0.62$ $V^2=2^2+2\times0.62\times8$ $V=3.73319\ldots$ so 3.73 m s^{-1} (3 s. f.)	M1 B1 B1 A1 B1 A1 B1 M1 B1 A1 M1 A1	W-E equation with KE, GPE and WD Change in KE (neglect sign) Either work done against resistance term (neglect sign) All terms correct (condone sign errors) All signs correct cao Use of N2L Weight component Appropriate *uvast* with **their** v_B and a cao	6
			total	14

11.

(a) (i)	$\hat{P}$ $Wb = Rx + Ta$	M1 A1	Moments equation attempted	
	$\uparrow$ $W = R + T$ Eliminate R	B1	Resolve or moments again	
	$Wb = (W - T)x + Ta$	M1	Substitute to eliminate R	
	so $x = \dfrac{Wb - Ta}{W - T}$	E1	Clearly shown	5
(ii)	**either** Need $x = 0$ giving $T = \dfrac{Wb}{a}$	B1	Both required	
	$R = W - \dfrac{Wb}{a} = \dfrac{W}{a}(a - b)$	B1	Either form [Award B2 for answer seen]	
	or Moments about edge through Q Result	M1 A1	Moments about edge through Q. R acting at edge thro' P	2
(b)		B1	All forces labelled correctly with arrows	
	$\uparrow$ $R = mg$	B1		
	$\rightarrow S = F$	B1		
	$\hat{A}$ $mgl\cos 72 = S \times 2l \sin 72$	M1 A1	Moments about a suitable point (if B used then F term receives the $S = F$ B1mark above)	
	so $F = \dfrac{mg\cos 72}{2\sin 72}$	A1	May be implied	
	$F = \mu R$ so $\mu = \dfrac{mg\cos 72}{2\sin 72} \div mg$	M1	Subst of **their** R and F required	
	$= \dfrac{1}{2}\cot 72$ or 0.162 (3 s. f.)	A1	Any form and any reasonable accuracy	8
			total	15

12.

(i)	$16\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=8\begin{pmatrix}1\\2\end{pmatrix}+4\begin{pmatrix}3\\3\end{pmatrix}+4\begin{pmatrix}6\\2\end{pmatrix}$	M1	Correct method	
		B1	Masses correct	
		A1	At least one c.m. correct (if x, y done separately, at least two c. m. cpts correct)	
	$\bar{x}=\frac{11}{4}$,	A1		
	$\bar{y}=\frac{9}{4}$	A1		5
(ii)	$(16+h)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\begin{pmatrix}44\\36\end{pmatrix}+h\begin{pmatrix}4\\2-\frac{h}{2}\end{pmatrix}$	M1	Correct method	
		B1	$16 + h$	
		M1	Attempt at $\begin{pmatrix}4\\2-\frac{h}{2}\end{pmatrix}$	
		A1	Correct	
	$\bar{x}=\frac{44+4h}{16+h}$	E1	Clearly shown	
	$\bar{y}=\frac{36+h\left(2-\frac{h}{2}\right)}{16+h}=\frac{72+h(4-h)}{2(16+h)}$	E1	Clearly shown	6
(iii)	B lies vertically below M so c. m. lies on BM which has equation $y=2-x$	E1	Full explanation but allow equation of BM not derived	
	$\frac{72+h(4-h)}{2(16+h)}=2-\frac{44+4h}{16+h}$	M1	Substituting for $\bar{x}$ and $\bar{y}$	
	$\Rightarrow 72+4h-h^2=64+4h-88-8h$			
	$\Rightarrow h^2-8h-96=0$	A1		
	giving $h = 14.6$ (3 s. f.)	A1	Accept only positive root given	4
				15

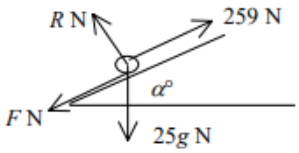
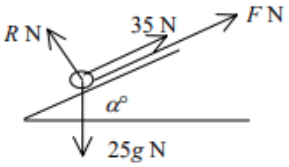
13.

(i)	[frequency] = T^{-1} [density] = $M L^{-3}$ [pressure] = [force] / [area] $= M L^{-1} T^{-2}$	B1 B1 B1 E1	Award for pressure as force/area seen	4
(ii)	$T^{-1} = L^{\alpha} (M L^{-1} T^{-2})^{\beta} (M L^{-3})^{\gamma}$ M: $0 = \beta + \gamma$ L: $0 = \alpha - \beta - 3\gamma$ T: $-1 = -2\beta$ giving $\alpha = -1$, $\beta = \frac{1}{2}$, $\gamma = -\frac{1}{2}$	M1 A1 M1 A1 A1 A1	Set up equation with all the terms Correct FT (i) Comparing powers for at least 1 dimension One equation correct. FT (i) Second equation correct. FT (i) All correct. cao	6
(iii)	$\frac{f}{f'} = \frac{ka'\sqrt{\frac{p}{d}}}{ka\sqrt{\frac{p'}{d'}}} = \frac{a'}{a}\sqrt{\frac{p}{p'}.\frac{d'}{d}}$ $a = a'$, $\frac{p}{p'} = 1.005$ and $\frac{d}{d'} = 0.98$ so $f = 440\sqrt{\frac{1.005}{0.98}}$ $= 445.576\ldots$ so 446 Hz (3 s. f.)	M1 B1 A1 A1	Attempt to eliminate k. Accept assigning values to a, a', p and d for full credit here and below. for either $\frac{p}{p'} = 1.005$ or $\frac{d}{d'} = 0.98$ or equivalents Seen or implied. FT (ii) only. Ignore wrong α used. cao	4
			total	14

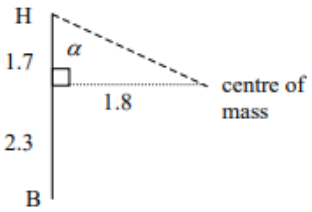
14.

(i)	Moments c.w. about A $80\times1.5=2.5R_C$ $R_C=48$ so 48 N	M1 A1 A1	Attempt at use of moments. Correct	3
(ii)	Moments c.w. about A $80\times1.5\times\sin 60=2.5R_C$ $R_C=24\sqrt{3}$ $=41.5692\ldots$ so 41.6 N (3 s. f.)	M1 B1 B1 A1	Attempt at moments equation and resolving LHS correct RHS correct	4
(iii)	Moments c.w. about A $80\times1.5\times\sin 60=2.5R_C+3\times20$ $R_C=24\left(\sqrt{3}-1\right)=17.569\ldots$ so 17.6 N (3 s. f.)	M1 A1	Equn in (ii) with 3×20 term attempted. cao	2
(iv)	$80\times1.5\times\sin 60=T\times3\sin 50$ $T=45.2206\ldots$ so 45.2 N (3 s. f.) $\uparrow\ Y+T\cos70=80$ $Y=64.533..$ so 64.5 N (3 s. f.) upwards	M1 A1 A1 M1 A1	Moments equation with all terms present and with attempt at resolution of T. Correct resolution of T Resolve vertically (or take moments again) with all forces present. cao. Accept no direction	5
			total	14

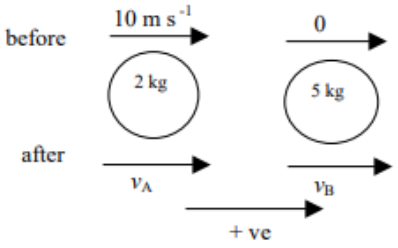
15.

(i)		B1	All forces present with arrows and labels. Angle not required	
		B1	All forces present with arrows and labels. Angle not required.	2
(ii)	up $259 = 25g\sin\alpha + F$	B1		
	down $F + 35 = 25g\sin\alpha$	B1		
	Eliminate F $259 = 50g\sin\alpha - 35$	M1	Eliminate one variable	
	so $\sin\alpha = 0.6$ and $\alpha = 36.869\ldots$ so 36.9 (3 s. f.)	A1		
	Back substitute $F = 25\times 9.8\times 0.6 - 35 = 112$	E1		5
(iii)	$F = \mu R$	M1	Must be substituted	
	$R = 25g\cos\alpha$	B1	FT **their** 0.8 from α	
	$\mu = \dfrac{112}{25\times 9.8\times 0.8} = \dfrac{112}{196} = \dfrac{4}{7}$ $(= 0.571428\ldots)$	A1	cao	3
(iv) (A)	The forces are not constant so acceleration is not constant. (Use of the method would have to be justified.)	E1		
(B)	$0.5\times 25\times v^2 = 25g\times 3\times 0.6 - 3\times 112$	M1 B1 B1 B1	W-E equation. All terms present KE term GPE term. FT **their** 0.6 from α WD against friction	
	$v^2 = 8.4$ so $v = 2.8982\ldots$ so 2.90 m s^{-1} (3s.f.)	A1	cao	6
			total	16

16.

(i)	$(16+4)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=16\begin{pmatrix}1\\ 2\end{pmatrix}+4\begin{pmatrix}5\\ 3.5\end{pmatrix}$ $\bar{x}=1.8$ $\bar{y}=2.3$	M1 B1 B1 A1 A1	Method for c.m. Correct masses Correct c.m. of one cpt or both x or both y cpts correct [Award up to 4/5 if other axes used]	5
(ii)	$\bar{z}=0$ ABHI XEFY $4\times\frac{1}{2}$ = $EX\times\frac{EX}{2}$ sub EX = 2 giving $2=2\times2/2$	M1 B1 B1 E1	Attempt at z cpt of c.m. of flaps to get $\bar{z}=0$ LHS RHS Justified [Award 2/4 for $4\times\frac{1}{2}=2\times\frac{EX}{2}\Rightarrow$ EX = 2]	4
(iii)	 $20\bar{x}=4\times0+12\times1.5+2\times4+2\times5=36$ $\bar{x}=1.8$ (the same) $\bar{y}$ remains 2.3	 M1 B1 A1 B1	FT use of wrong axes Appropriate method to deal with flaps Any non-zero component of $\bar{x}$ correct [Award M1 A0 A1 for 'the same' stated WW]	4
(iv)	 $\tan\alpha=\frac{\bar{x}}{4-\bar{y}}$ α = 46.636… so 46.6° (3 s. f.)	 B1 M1 A1	 Identify required angle and relate to $(\bar{x},\bar{y})$. May be implied by correct expression for α. Use of arctan expression. Accept only correct expression or $\tan\alpha=\frac{\bar{x}}{\bar{y}}$ or $\tan\alpha=\frac{\bar{y}}{\bar{x}}$, using **their** values. cao. Any reasonable accuracy	3
			total	16

17.

(i)	$Ft = 4\times5 = 20$ N s $20 = 2v$ so $v = 10$	 E1		1
(ii)	 PCLM $20 = 2v_A + 5\,v_B$ NEL $\dfrac{v_B - v_A}{0-10} = -0.54$ Either solve or substitute values in **both** equations to give $v_A = -1$ and $v_B = 4.4$	 M1 A1 M1 A1 E1	 Use of PCLM Any form Use of NEL Any form Directions must be clearly established (accept diagram).	5
(iii)	$\leftarrow$ $5(2.2-(-4.4))$ $= 33$ N s	M1 A1	$I = m(v-u)$ attempted Accept ± 33 N s	2
(iv) (A)	NEL $\dfrac{v_B + 1}{-10} = -e$ so $v_B = 10e - 1$ PCLM $20 = -2 + mv_B$ so $v_B = \dfrac{22}{m}$ Eliminating v_B gives $e = \dfrac{22+m}{10m}$	M1 M1 E1	Use of NEL. Allow only sign errors Use of PCLM. Allow only sign errors Clearly shown	
(B)	Need $v_B > 2$ so $m < 11$ Hence $e > \dfrac{\frac{22}{11}+1}{10} = 0.3$ so $0.3 < e \leq 1$	E1 M1 E1	Argue that $m < 11$ Reorganise inequality to show result (accept use of =) Clearly shown (the inequality must be justified) $e \leq 1$ is not required	6
			total	14