Groups Past Paper Question Pack 2

MEI P6 Jun 02

The set $G = \{1, 3, 7, 9, 11, 13, 17, 19\}$ is a group under the binary operation of multiplication modulo 20.

(i) Give the combination table for G. [4]

(ii) State the inverse of each element of G. [2]

(iii) Find the order of each element of G. [2]

(iv) List all the subgroups of G.

Identify those subgroups which are isomorphic to one another. [7]

(v) For each of the following, state, giving reasons, whether or not the given set and binary operation is a group. If it is a group, state, giving a reason, whether or not it is isomorphic to G.

(A) $J = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under multiplication modulo 8

(B) $K = \{0, 1, 2, 3, 4, 5, 6, 7\}$ under addition modulo 8 [5]

MEI P6 Jun 04

The set G consists of all real numbers not equal to 2.

A binary operation $*$ is defined on real numbers x, y by $x * y = xy - 2x - 2y + 6$.

(i) Prove that G, with the binary operation $*$, is a group. [9]

(ii) Find an element of G of order 2. [2]

The set $H = \{3, 5, 9, 11\}$ has a binary operation $\circ$ defined by

$x \circ y$ is the remainder when $x * y$ is divided by 20.

(iii) Give the combination table for H, and hence prove that H is a group. [5]

(iv) Determine whether H is a cyclic group or not. [2]

(v) Explain why H is not a subgroup of G. [2]

MEI P6 Jun 05

An abelian group $G = \{a, b, c, d, e, f, g, h\}$ has the following composition table.

	a	b	c	d	e	f	g	h
a	b	c	e	f	a	g	h	d
b	c	e	a	g	b	h	d	f
c	e	a	b	h	c	d	f	g
d	f	g	h	e	d	a	b	c
e	a	b	c	d	e	f	g	h
f	g	h	d	a	f	b	c	e
g	h	d	f	b	g	c	e	a
h	d	f	g	c	h	e	a	b

(i) State the inverse of each element of G. [2]

(ii) Find the order of each element of G. [3]

(iii) List all the proper subgroups of G. [5]

OCR FP3 Jan 06

1.

The tables shown below are the operation tables for two isomorphic groups G and H.

G	a	b	c	d
a	d	a	b	c
b	a	b	c	d
c	b	c	d	a
d	c	d	a	b

H	2	4	6	8
2	4	8	2	6
4	8	6	4	2
6	2	4	6	8
8	6	2	8	4

(i) For each group, state the identity element and list the elements of any proper subgroups. [4]

(ii) Establish the isomorphism between G and H by showing which elements correspond. [3]

2.

A group G has an element a with order n, so that $a^n = e$, where e is the identity. It is given that x is any element of G distinct from a and e.

(i) Prove that the order of $x^{-1}ax$ is n, making it clear which group property is used at each stage of your proof. [6]

(ii) Express the inverse of $x^{-1}ax$ in terms of some or all of x, x^{-1}, a and a^{-1}, showing sufficient working to justify your answer. [3]

(iii) It is now given that a commutes with every element of G. Prove that a^{-1} also commutes with every element. [2]

OCR FP3 Jun 06

1.

(a) For the infinite group of non-zero complex numbers under multiplication, state the identity element and the inverse of $1 + 2i$, giving your answers in the form $a + ib$. [3]

(b) For the group of matrices of the form $\begin{pmatrix} a & 0 \\ 0 & 0 \end{pmatrix}$ under matrix addition, where $a \in \mathbb{R}$, state the identity element and the inverse of $\begin{pmatrix} 3 & 0 \\ 0 & 0 \end{pmatrix}$. [2]

2.

A group D of order 10 is generated by the elements a and r, with the properties $a^2 = e$, $r^5 = e$ and $r^4 a = ar$, where e is the identity. Part of the operation table is shown below.

	e	a	r	r^2	r^3	r^4	ar	ar^2	ar^3	ar^4
e	e	a	r	r^2	r^3	r^4	ar	ar^2	ar^3	ar^4
a	a	e	ar	ar^2	ar^3	ar^4				
r	r		r^2	r^3	r^4	e				
r^2	r^2		r^3	r^4	e	r				
r^3	r^3		r^4	e	r	r^2				
r^4	r^4	ar	e	r	r^2	r^3				
ar	ar		ar^2	ar^3	ar^4	a				
ar^2	ar^2		ar^3	ar^4	a	ar		**E**		
ar^3	ar^3		ar^4	a	ar	ar^2				
ar^4	ar^4		a	ar	ar^2	ar^3				

(i) Give a reason why D is not commutative. [1]

(ii) Write down the orders of any possible proper subgroups of D. [2]

(iii) List the elements of a proper subgroup which contains

(a) the element a, [1]

(b) the element r. [1]

(iv) Determine the order of each of the elements r^3, ar and ar^2. [4]

(v) Copy and complete the section of the table marked **E**, showing the products of the elements ar, ar^2, ar^3 and ar^4. [5]

OCR FP3 Jan 07

1.

(i) Show that the set of numbers $\{3, 5, 7\}$, under multiplication modulo 8, does not form a group. [2]

(ii) The set of numbers $\{3, 5, 7, a\}$, under multiplication modulo 8, forms a group. Write down the value of a. [1]

(iii) State, justifying your answer, whether or not the group in part **(ii)** is isomorphic to the multiplicative group $\{e, r, r^2, r^3\}$, where e is the identity and $r^4 = e$. [2]

2.

A multiplicative group G of order 9 has distinct elements p and q, both of which have order 3. The group is commutative, the identity element is e, and it is given that $q \neq p^2$.

(i) Write down the elements of a proper subgroup of G

(a) which does not contain q, [1]

(b) which does not contain p. [1]

(ii) Find the order of each of the elements pq and pq^2, justifying your answers. [3]

(iii) State the possible order(s) of proper subgroups of G. [1]

(iv) Find two proper subgroups of G which are distinct from those in part **(i)**, simplifying the elements. [4]

OCR FP3 Jun 07

1.

Elements of the set $\{p, q, r, s, t\}$ are combined according to the operation table shown below.

	p	q	r	s	t
p	t	s	p	r	q
q	s	p	q	t	r
r	p	q	r	s	t
s	r	t	s	q	p
t	q	r	t	p	s

(i) Verify that $q(st) = (qs)t$. [2]

(ii) Assuming that the associative property holds for all elements, prove that the set $\{p, q, r, s, t\}$, with the operation table shown, forms a group G. [4]

(iii) A multiplicative group H is isomorphic to the group G. The identity element of H is e and another element is d. Write down the elements of H in terms of e and d. [2]

2.

The set S consists of the numbers 3^n, where $n \in \mathbb{Z}$. ($\mathbb{Z}$ denotes the set of integers $\{0, \pm1, \pm2, \ldots\}$.)

(i) Prove that the elements of S, under multiplication, form a commutative group G. (You may assume that **addition** of integers is associative and commutative.) [6]

(ii) Determine whether or not each of the following subsets of S, under multiplication, forms a subgroup of G, justifying your answers.

(a) The numbers 3^{2n}, where $n \in \mathbb{Z}$. [2]

(b) The numbers 3^n, where $n \in \mathbb{Z}$ and $n \geqslant 0$. [2]

(c) The numbers $3^{(\pm n^2)}$, where $n \in \mathbb{Z}$. [2]

OCR FP3 Jan 08

1.

(a) A group G of order 6 has the combination table shown below.

	e	a	b	p	q	r
e	e	a	b	p	q	r
a	a	b	e	r	p	q
b	b	e	a	q	r	p
p	p	q	r	e	a	b
q	q	r	p	b	e	a
r	r	p	q	a	b	e

(i) State, with a reason, whether or not G is commutative. [1]

(ii) State the number of subgroups of G which are of order 2. [1]

(iii) List the elements of the subgroup of G which is of order 3. [1]

(b) A multiplicative group H of order 6 has elements e, c, c^2, c^3, c^4, c^5, where e is the identity. Write down the order of each of the elements c^3, c^4 and c^5. [3]

2.

Groups A, B, C and D are defined as follows:

A: the set of numbers $\{2, 4, 6, 8\}$ under multiplication modulo 10,

B: the set of numbers $\{1, 5, 7, 11\}$ under multiplication modulo 12,

C: the set of numbers $\{2^0, 2^1, 2^2, 2^3\}$ under multiplication modulo 15,

D: the set of numbers $\left\{\frac{1+2m}{1+2n}, \text{where } m \text{ and } n \text{ are integers}\right\}$ under multiplication.

(i) Write down the identity element for each of groups A, B, C and D. [2]

(ii) Determine in each case whether the groups

A and B,

B and C,

A and C

are isomorphic or non-isomorphic. Give sufficient reasons for your answers. [5]

(iii) Prove the closure property for group D. [4]

(iv) Elements of the set $\left\{\frac{1+2m}{1+2n}, \text{where } m \text{ and } n \text{ are integers}\right\}$ are combined under **addition**. State which of the four basic group properties are **not** satisfied. (Justification is not required.) [2]

OCR FP3 Jun 08

1.

(a) A cyclic multiplicative group G has order 12. The identity element of G is e and another element is r, with order 12.

(i) Write down, in terms of e and r, the elements of the subgroup of G which is of order 4. [2]

(ii) Explain briefly why there is no proper subgroup of G in which two of the elements are e and r. [1]

(b) A group H has order mnp, where m, n and p are prime. State the possible orders of proper subgroups of H. [2]

2.

The operation $\circ$ on real numbers is defined by $a \circ b = a|b|$.

(i) Show that $\circ$ is not commutative. [2]

(ii) Prove that $\circ$ is associative. [4]

(iii) Determine whether the set of real numbers, under the operation $\circ$, forms a group. [4]