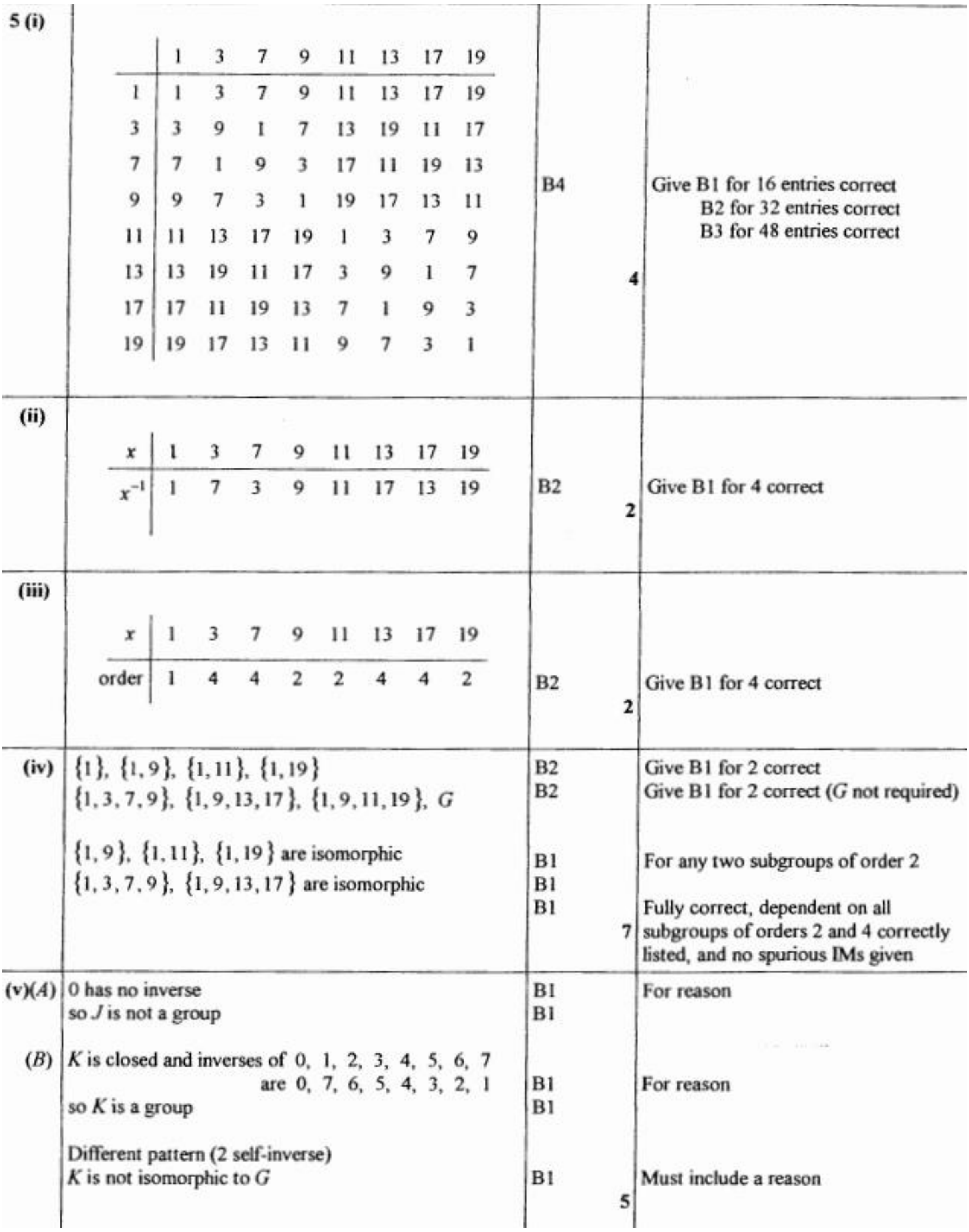
Groups Past Paper Question Pack 2 – Mark Schemes

MEI P6 Jun 02



5 (i)

	1	3	7	9	11	13	17	19
1	1	3	7	9	11	13	17	19
3	3	9	1	7	13	19	11	17
7	7	1	9	3	17	11	19	13
9	9	7	3	1	19	17	13	11
11	11	13	17	19	1	3	7	9
13	13	19	11	17	3	9	1	7
17	17	11	19	13	7	1	9	3
19	19	17	13	11	9	7	3	1

B4 4

Give B1 for 16 entries correct
B2 for 32 entries correct
B3 for 48 entries correct

(ii)

x	1	3	7	9	11	13	17	19
x^{-1}	1	7	3	9	11	17	13	19

B2 2

Give B1 for 4 correct

(iii)

x	1	3	7	9	11	13	17	19
order	1	4	4	2	2	4	4	2

B2 2

Give B1 for 4 correct

(iv)

$\{1\}$, $\{1,9\}$, $\{1,11\}$, $\{1,19\}$	B2	Give B1 for 2 correct
$\{1,3,7,9\}$, $\{1,9,13,17\}$, $\{1,9,11,19\}$, G	B2	Give B1 for 2 correct (G not required)
$\{1,9\}$, $\{1,11\}$, $\{1,19\}$ are isomorphic	B1	For any two subgroups of order 2
$\{1,3,7,9\}$, $\{1,9,13,17\}$ are isomorphic	B1	
	B1 7	Fully correct, dependent on all subgroups of orders 2 and 4 correctly listed, and no spurious IMs given

(v)(*A*)

0 has no inverse	B1	For reason
so J is not a group	B1	

(*B*)

K is closed and inverses of 0, 1, 2, 3, 4, 5, 6, 7 are 0, 7, 6, 5, 4, 3, 2, 1	B1	For reason
so K is a group	B1	
Different pattern (2 self-inverse) K is not isomorphic to G	B1 5	Must include a reason

<table>
<tr><td>5 (i)</td><td>G is closed if $x * y \neq 2$
$x * y = 2 \;\Rightarrow\; (x-2)(y-2) = 0$</td><td>M1</td><td></td></tr>
<tr><td></td><td>As $x \neq 2$ and $y \neq 2$, we have $x * y \neq 2$</td><td>A1</td><td></td></tr>
<tr><td></td><td>$(x * y) * z = (xy - 2x - 2y + 6) * z$
$= (xy - 2x - 2y + 6)z - 2(xy - 2x - 2y + 6) - 2z + 6$</td><td>M1</td><td>M0 if only particular example(s) given</td></tr>
<tr><td></td><td>$= xyz - 2xz - 2yz - 2xy + 4x + 4y + 4z - 6$</td><td>A1</td><td></td></tr>
<tr><td></td><td>$x * (y * z)$
$= x(yz - 2y - 2z + 6) - 2x - 2(yz - 2y - 2z + 6) + 6$
$= xyz - 2xy - 2xz - 2yz + 4x + 4y + 4z - 6$
Hence G is associative</td><td>A1</td><td></td></tr>
<tr><td></td><td>The identity element is 3
(since $3 * x = 3x - 6 - 2x + 6 = x$)</td><td>B1</td><td></td></tr>
<tr><td></td><td>$x * y = 3 \;\Leftrightarrow\; y = \dfrac{2x-3}{x-2}$</td><td>M1A1</td><td></td></tr>
<tr><td></td><td>Since $x \neq 2$ and $\dfrac{2x-3}{x-2} \neq 2$, every element of G has an inverse in G</td><td>A1
9</td><td></td></tr>
<tr><td>(ii)</td><td>$x * x = 3 \;\Leftrightarrow\; x^2 - 4x + 6 = 3$
$\Leftrightarrow\; x = 1,\; 3$</td><td>M1</td><td></td></tr>
<tr><td></td><td>The only element with order 2 is 1</td><td>A1
2</td><td></td></tr>
<tr><td>(iii)</td><td><table>
<tr><td></td><td>3</td><td>5</td><td>9</td><td>11</td></tr>
<tr><td>3</td><td>3</td><td>5</td><td>9</td><td>11</td></tr>
<tr><td>5</td><td>5</td><td>11</td><td>3</td><td>9</td></tr>
<tr><td>9</td><td>9</td><td>3</td><td>11</td><td>5</td></tr>
<tr><td>11</td><td>11</td><td>9</td><td>5</td><td>3</td></tr>
</table></td><td>B2</td><td>Give B1 for one bold value correct</td></tr>
<tr><td></td><td>Table shows H is closed
$\circ$ is associative since $*$ is associative
The identity element is 3</td><td>B1</td><td>For any two of these statements</td></tr>
<tr><td></td><td>3, 5, 9, 11 have inverses 3, 9, 5, 11</td><td>B2
5</td><td>Give B1 for two correct</td></tr>
<tr><td>(iv)</td><td>H is cyclic since it has an element of order 4</td><td>M1</td><td></td></tr>
<tr><td></td><td>The element 5 (or 9) has order 4</td><td>A1
2</td><td></td></tr>
<tr><td>(v)</td><td>H is not a subgroup of G since the binary operation is different;</td><td>M1</td><td></td></tr>
<tr><td></td><td>e.g. in G, $5 * 9 = 23$; but in H, $5 \circ 9 = 3$</td><td>A1
2</td><td></td></tr>
</table>

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5(a)(i)	Element a b c d e f g h Inverse c b a d e h g f	B2 **2**	Give B1 for five correct
(ii)	Element a b c d e f g h Order 4 2 4 2 1 4 2 4	B3 **3**	Give B2 for six correct B1 for three correct
(iii)	$\{e, b\}$, $\{e, d\}$, $\{e, g\}$ $\{e, a, b, c\}$ $\{e, b, f, h\}$ $\{e, b, d, g\}$	B2 B1 B1 B1 **5**	Give B1 for two correct If more than 6 subgroups given, deduct B1 from total for each in excess of 6 (but ignore $\{e\}$ and G)

OCR FP3 Jan 06

1.

(i) Identities $b,\ 6$	B1 B1	For correct identities
Subgroups $\{b, d\},\ \{6, 4\}$	B1 B1	For correct subgroups
	4	
(ii) $\{a, b, c, d\} \leftrightarrow \{2, 6, 8, 4\}$ or $\{8, 6, 2, 4\}$	B1 B1	For $b \leftrightarrow 6,\ d \leftrightarrow 4$
	B1 **3**	For $a, c \leftrightarrow 2, 8$ in either order
		SR If B0 B0 B0 then M1 A1 may be awarded for stating the orders of all elements in G and H
	$\boxed{7}$	

2.

(i) $(x^{-1}ax)^m = (x^{-1}ax)(x^{-1}ax)\dots(x^{-1}ax)$	M1	For considering powers of $x^{-1}ax$
$= x^{-1}aa\dots ax$, associativity, $xx^{-1} = e$	A1 A1	For using associativity and inverse properties
$= x^{-1}a^m x = x^{-1}ex$ when $m = n$, not $m < n$	B1	For using order of a correctly
$= x^{-1}x$	A1	For using property of identity
$= e \Rightarrow$ order n	A1 **6**	For correct conclusion
(ii) ***EITHER*** $(x^{-1}ax)z = e$	M1	For attempt to solve for z **AEF**
$\Rightarrow axz = xe = x \Rightarrow xz = a^{-1}x$	A1	For using pre- or post multiplication
$\Rightarrow z = x^{-1}a^{-1}x$	A1	For correct answer
OR Use $(pq)^{-1} = q^{-1}p^{-1}$ *OR* $(pqr)^{-1} = r^{-1}q^{-1}p^{-1}$	M1	For applying inverse of a product of elements
State $(x^{-1})^{-1} = x$	A1	For stating this property
Obtain $x^{-1}a^{-1}x$	A1 **3**	For correct answer with no incorrect working **SR** correct answer with no working scores B1 only
(iii) $ax = xa \Rightarrow x = a^{-1}xa$	M1	Start from commutative property for ax
$\Rightarrow xa^{-1} = a^{-1}x$	A1 **2**	Obtain commutative property for $a^{-1}x$
	$\boxed{11}$	

OCR FP3 Jun 06

1.

(a) Identity $= 1+0\text{i}$	B1		For correct identity. Allow 1
Inverse $= \dfrac{1}{1+2\text{i}}$	B1		For $\dfrac{1}{1+2\text{i}}$ seen or implied
$= \dfrac{1}{1+2\text{i}} \times \dfrac{1-2\text{i}}{1-2\text{i}} = \frac{1}{5} - \frac{2}{5}\text{i}$	B1	3	For correct inverse **AEFcartesian**
(b) Identity $= \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$	B1		For correct identity
Inverse $= \begin{pmatrix} -3 & 0 \\ 0 & 0 \end{pmatrix}$	B1	2	For correct inverse
		$\boxed{5}$	

2.

(i) $r^4 . a \neq a . r^4$	B1	1	For stating the non-commutative product in the given table, or justifying another correct one
(ii) Possible subgroups order 2, 5	B1		For either order stated
	B1	2	For both orders stated, and no more (Ignore 1)
(iii) **(a)** $\{e, a\}$	B1		For correct subgroup
(b) $\{e, r, r^2, r^3, r^4\}$	B1	2	For correct subgroup
(iv) order of $r^3 = 5$	B1		For correct order
$(ar)^2 = ar.ar = r^4 a.ar = e$	M1		For attempt to find $(ar)^m = e$ *OR* $(ar^2)^m = e$
$\Rightarrow$ order of $ar = 2$	A1		For correct order
$(ar^2)^2 = ar^2 ar.r = ar^2 r^4 a.r = ara.r = e$			
$\Rightarrow$ order of $ar^2 = 2$	A1	4	For correct order
(v) $\begin{array}{c\|cccc} & ar & ar^2 & ar^3 & ar^4 \\ \hline ar & e & r & r^2 & r^3 \\ ar^2 & r^4 & e & r & r^2 \\ ar^3 & r^3 & r^4 & e & r \\ ar^4 & r^2 & r^3 & r^4 & e \end{array}$			If the border elements ar ar^2 ar^3 ar^4 are not written, it will be assumed that the products arise from that order
	B1		For all 16 elements of the form e or r^m
	B1		For all 4 elements in leading diagonal $= e$
	B1		For no repeated elements in any completed row or column
	B1		For any two rows or columns correct
	B1	5	For all elements correct
		$\boxed{14}$	

OCR FP3 Jan 07

1.

(i) Attempt to show no closure	M1	For showing operation table or otherwise
$3\times3=1,\ 5\times5=1$ *OR* $7\times7=1$	A1	For a convincing reason
OR Attempt to show no identity	M1	For attempt to find identity *OR* for showing operation table
Show $a\times e=a$ has no solution	A1 **2**	For showing identity is not 3, not 5, and not 7 by reference to operation table or otherwise
(ii) $(a=)\ 1$	B1 **1**	For value of a stated
(iii) *EITHER*: $\{e,r,r^2,r^3\}$ is cyclic, **(ii)** group is not cyclic	B1*	For a pair of correct statements
OR: $\{e,r,r^2,r^3\}$ has 2 self-inverse elements, **(ii)** group has 4 self-inverse elements	B1*	For a pair of correct statements
OR: $\{e,r,r^2,r^3\}$ has 1 element of order 2 **(ii)** group has 3 elements of order 2	B1*	For a pair of correct statements
OR: $\{e,r,r^2,r^3\}$ has element(s) of order 4 **(ii)** group has no element of order 4	B1*	For a pair of correct statements
Not isomorphic	B1 (dep*) **2** $\boxed{5}$	For correct conclusion

2.

(i) **(a)** $e,\ p,\ p^2$	B1	For correct elements
(b) $e,\ q,\ q^2$	B1 **2**	For correct elements **SR** If the answers to parts **(i)** and **(iv)** are reversed, full credit may be earned for both parts
(ii) $p^3=q^3=e\Rightarrow(pq)^3=p^3q^3=e$	M1	For finding $(pq)^3$ or $(pq^2)^3$
$\Rightarrow$ order 3	A1	For correct order
$(pq^2)^3=p^3q^6=p^3(q^3)^2=e\Rightarrow$ order 3	A1 **3**	For correct order **SR** For answer(s) only allow B1 for either or both
(iii) 3	B1 **1**	For correct order and no others
(iv)	B1	For stating e and either pq or p^2q^2
$e,\ pq,\ p^2q^2$ *OR* $e,\ pq,\ (pq)^2$	B1	For all 3 elements and no more
	B1	For stating e and either pq^2 or p^2q
$e,\ pq^2,\ p^2q$ *OR* $e,\ pq^2,(pq^2)^2$ *OR* $e,\ p^2q,(p^2q)^2$	B1 **4** $\boxed{10}$	For all 3 elements and no more

OCR FP3 Jun 07

1.

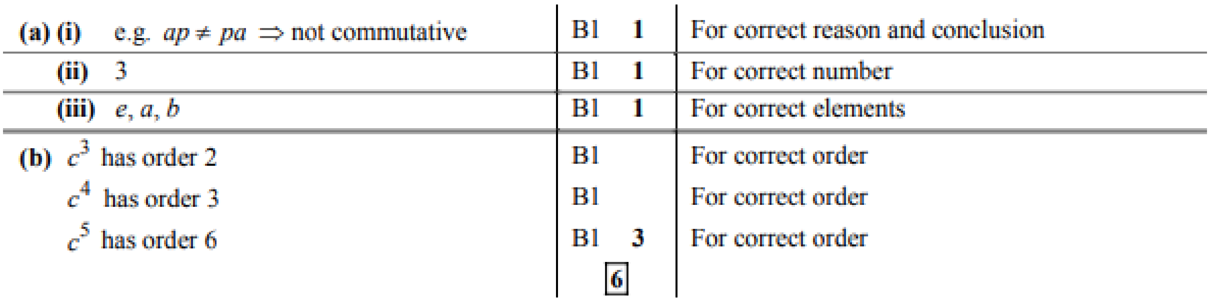
(i) $q(st) = qp = s$	B1	For obtaining s
$(qs)t = tt = s$	B1 **2**	For obtaining s
(ii) METHOD 1		
Closed: see table	B1	For stating closure with reason
Identity = r	B1	For stating identity r
Inverses: $p^{-1} = s,\ q^{-1} = t,\ (r^{-1} = r),$	M1	For checking for inverses
$s^{-1} = p,\ t^{-1} = q$	A1 **4**	For stating inverses *OR* For giving sufficient explanation to justify each element has an inverse eg r occurs once in each row and/or column
(iii) e, d, d^2, d^3, d^4	B2 **2**	For stating all elements **AEF** eg d^{-1}, d^{-2}, dd
	$\boxed{\mathbf{8}}$	

2.

9 **(i)** $3^n \times 3^m = 3^{n+m},\ n + m \in Z$	B1	For showing closure
$\left(3^p \times 3^q\right) \times 3^r = \left(3^{p+q}\right) \times 3^r = 3^{p+q+r}$	M1	For considering 3 distinct elements, seen bracketed 2+1 or 1+2
$= 3^p \times \left(3^{q+r}\right) = 3^p \times \left(3^q \times 3^r\right) \Rightarrow$ associativity	A1	For correct justification of associativity
Identity is 3^0	B1	For stating identity. Allow 1
Inverse is 3^{-n}	B1	For stating inverse
$3^n \times 3^m = 3^{n+m} = 3^{m+n} = 3^m \times 3^n \Rightarrow$ commutativity	B1 **6**	For showing commutativity
(ii) (a) $3^{2n} \times 3^{2m} = 3^{2n+2m} \left(= 3^{2(n+m)}\right)$	B1*	For showing closure
Identity, inverse OK	B1 (*dep) **2**	For stating other two properties satisfied and hence a subgroup
(b) For 3^{-n},	M1	For considering inverse
$-n \notin$ subset	A1 **2**	For justification of not being a subgroup
		3^{-n} must be seen here or in **(i)**
(c) *EITHER*: eg $3^{1^2} \times 3^{2^2} = 3^5$	M1	For attempting to find a specific counter-example of closure
$\neq 3^{r^2} \Rightarrow$ not a subgroup	A1 **2**	For a correct counter-example and statement that it is not a subgroup
OR: $3^{n^2} \times 3^{m^2} = 3^{n^2+m^2}$	M1	For considering closure in general
$\neq 3^{r^2}$ eg $1^2 + 2^2 = 5 \Rightarrow$ not a subgroup	A1	For explaining why $n^2 + m^2 \neq r^2$ in general and statement that it is not a subgroup
	$\boxed{\mathbf{12}}$	

OCR FP3 Jan 08

1.



(a) (i)	e.g. $ap \neq pa \Rightarrow$ not commutative	B1	**1**	For correct reason and conclusion
(ii)	3	B1	**1**	For correct number
(iii)	e, a, b	B1	**1**	For correct elements
(b)	c^3 has order 2	B1		For correct order
	c^4 has order 3	B1		For correct order
	c^5 has order 6	B1	**3**	For correct order
			$\boxed{6}$	

2.

<table>
<tr><td>(i) Group A: $e = 6$
Group B: $e = 1$
Group C: $e = 2^0$ <i>OR</i> 1
Group D: $e = 1$</td><td>B1
B1
2</td><td>For any two correct identities
For two other correct identities
AEF for D, but not “$m = n$”</td></tr>
<tr><td>(ii) <i>EITHER</i> <i>OR</i>
$\begin{array}{c|cccc} A & 2 & 4 & 6 & 8 \\ \hline 2 & 4 & 8 & 2 & 6 \\ 4 & 8 & 6 & 4 & 2 \\ 6 & 2 & 4 & 6 & 8 \\ 8 & 6 & 2 & 8 & 4 \end{array}$ orders of elements 1, 2, 4, 4 <i>OR</i> cyclic group
$\begin{array}{c|cccc} B & 1 & 5 & 7 & 11 \\ \hline 1 & 1 & 5 & 7 & 11 \\ 5 & 5 & 1 & 11 & 7 \\ 7 & 7 & 11 & 1 & 5 \\ 11 & 11 & 7 & 5 & 1 \end{array}$ orders of elements 1, 2, 2, 2 <i>OR</i> non-cyclic group <i>OR</i> Klein group
$\begin{array}{c|cccc} C & 2^0 & 2^1 & 2^2 & 2^3 \\ \hline 2^0 & 2^0 & 2^1 & 2^2 & 2^3 \\ 2^1 & 2^1 & 2^2 & 2^3 & 2^0 \\ 2^2 & 2^2 & 2^3 & 2^0 & 2^1 \\ 2^3 & 2^3 & 2^0 & 2^1 & 2^2 \end{array}$ orders of elements 1, 2, 4, 4 <i>OR</i> cyclic group</td><td>B1*
B1*</td><td>For showing group table
<i>OR</i> sufficient details of orders of elements
<i>OR</i> stating cyclic / non-cyclic / Klein group
(as appropriate)
for one of groups A, B, C
for another of groups A, B, C</td></tr>
<tr><td>$A \not\cong B$
$B \not\cong C$
$A \cong C$</td><td>B1
(dep*)
B1
(dep*)
B1
(dep*)
5</td><td>For stating non-isomorphic
For stating non-isomorphic
For stating isomorphic
with sufficient detail relating to the first 2 mark</td></tr>
<tr><td>(iii) $\frac{1+2m}{1+2n} \times \frac{1+2p}{1+2q} = \frac{1+2m+2p+4mp}{1+2n+2q+4nq}$
$= \frac{1+2(m+p+2mp)}{1+2(n+q+2nq)} \equiv \frac{1+2r}{1+2s}$</td><td>M1*
M1
(dep*)
A1
A1 4</td><td>For considering product of 2 distinct elements of this form
For multiplying out
For simplifying to form shown
For identifying as correct form, so closed
SR $\frac{\text{odd}}{\text{odd}} \times \frac{\text{odd}}{\text{odd}} = \frac{\text{odd}}{\text{odd}}$ earns full credit
SR If clearly attempting to prove commutativity, allow at most M1</td></tr>
<tr><td>(iv) Closure not satisfied
Identity and inverse not satisfied</td><td>B1
B1 2
$\boxed{13}$</td><td>For stating closure
For stating identity and inverse
SR If associativity is stated as not satisfied, then award at most B1 B0 <i>OR</i> B0 B1</td></tr>
</table>

OCR FP3 Jun 08

1.

(a)(i)	e, r^3, r^6, r^9	M1		For stating e, r^m (any m ..**2**), and 2 other different elements in terms of e and r
		A1	**2**	For all elements correct
(ii)	r generates G	B1	**1**	For this or any statement equivalent to: all elements of G are included in a group with e and r *OR* order of $r >$ order of all possible proper subgroups
(b)	m, n, p, mn, np, pm	B1		For any 3 orders correct
		B1	**2**	For all 6 correct and no extras (Ignore 1 and mnp)
		$\boxed{5}$		

2.

(i)	When a, b have opposite signs,	M1		For considering sign of $a\|b\|$ *OR* $b\|a\|$ in general or in a specific case
	$a\|b\| = \pm ab$, $b\|a\| = \mp ba \;\Rightarrow\; a\|b\| \neq b\|a\|$	A1	**2**	For showing that $a\|b\| \neq b\|a\|$ Note that $\|x\| = \sqrt{x^2}$ may be used
(ii)	$(a \circ b) \circ c = (a\|b\|) \circ c = a\|b\|\|c\|$ *OR* $a\|bc\|$	M1		For using 3 distinct elements and simplifying $(a \circ b) \circ c$ *OR* $a \circ (b \circ c)$
		A1		For obtaining correct answer
	$a \circ (b \circ c) = a \circ (b\|c\|) = a\|b\|c\|\| = a\|b\|\|c\|$ *OR* $a\|bc\|$	M1		For simplifying the other bracketed expression
		A1	**4**	For obtaining the same answer
(iii)		B1*		For stating $e = \pm 1$ *OR* no identity
	EITHER $a \circ e = a\|e\| = a \;\Rightarrow\; e = \pm 1$	M1		For attempting algebraic justification of +1 and –1 for e
	OR $e \circ a = e\|a\| = a$ $\Rightarrow e = 1$ for $a > 0$, $e = -1$ for $a < 0$	A1		For deducing no (unique) identity
	Not a group	B1 (*dep)	**4**	For stating not a group
		$\boxed{10}$		