Groups Past Paper Question Pack – Mark Schemes

MEI FP3 2006

4 (i)

	I	J	K	L	–I	–J	–K	–L
I	I	J	K	L	–I	–J	–K	–L
J	J	–I	L	–K	–J	I	–L	K
K	K	–L	–I	**J**	**–K**	**L**	**I**	**–J**
L	L	K	**–J**	**–I**	**–L**	**–K**	**J**	**I**
–I	–I	–J	**–K**	**–L**	**I**	**J**	**K**	**L**
–J	–J	I	**–L**	**K**	**J**	**–I**	**L**	**–K**
–K	–K	L	**I**	**–J**	**K**	**–L**	**–I**	**J**
–L	–L	–K	**J**	**I**	**L**	**K**	**–J**	**–I**

B6 — **6**

Give B5 for 30 (bold) entries correct
Give B4 for 24 (bold) entries correct
Give B3 for 18 (bold) entries correct
Give B2 for 12 (bold) entries correct
Give B1 for 6 (bold) entries correct

(ii)

Element	I	J	K	L	–I	–J	–K	–L
Inverse	**I**	**–J**	**–K**	**–L**	**–I**	**J**	**K**	**L**

B3 — **3**

Give B2 for six correct
Give B1 for three correct

(iii)

Element	I	J	K	L	–I	–J	–K	–L
Order	1	4	4	4	2	4	4	4

B3 — **3**

Give B2 for six correct
Give B1 for three correct

(iv)

Only two elements of *G* do not have order 4; so any subgroup of order 4 must contain an element of order 4 — M1A1

A subgroup of order 4 is cyclic if it contains an element of order 4 — B1 (may be implied)

Hence any subgroup of order 4 is cyclic — A1 For completion

4

OR If a group of order 4 is not cyclic,
it contains three elements of order 2 B1
G has only one element of order 2; so this cannot occur M1A1
So any subgroup of order 4 is cyclic A1

(v)

$\{\mathbf{I}, -\mathbf{I}\}$ — B1
$\{\mathbf{I}, \mathbf{J}, -\mathbf{I}, -\mathbf{J}\}$ — B1
$\{\mathbf{I}, \mathbf{K}, -\mathbf{I}, -\mathbf{K}\}$ — B1
$\{\mathbf{I}, \mathbf{L}, -\mathbf{I}, -\mathbf{L}\}$ — B1

B1 — **5** — For $\{\mathbf{I}, -\mathbf{I}\}$, at least one correct subgroup of order 4, and no wrong subgroups. This mark is lost if *G* or $\{\mathbf{I}\}$ is included

4 (i)	By Lagrange's theorem, a proper subgroup has order 2 or 5 A group of prime order is cyclic Hence every proper subgroup is cyclic	M1 A1 M1 A1 **4**	Using Lagrange *(need not be mentioned explicitly)* or equivalent For completion
(ii)	e.g. $2^2=4,\ 2^3=8,\ 2^4=5,\ 2^5=10,$ $2^6=9,\ 2^7=7,\ 2^8=3,\ 2^9=6,\ 2^{10}=1$ 2 has order 10, hence M is cyclic	M1 A1 A1 A1 **4**	Considering order of an element Identifying an element of order 10 (2, 6, 7 or 8) Fully justified For conclusion (can be awarded after M1A1A0)
(iii)	$\{1,\ 10\}$ $\{1,\ 3,\ 4,\ 5,\ 9\}$	B1 B2 **3**	Ignore $\{1\}$ and M Deduct 1 mark (from B1B2) for each (proper) subgroup given in excess of 2
(iv)	E is the identity A, C, G, I are rotations B, D, F, H, J are reflections	B1 M1 A1 A1 **4**	Considering elements of order 2 *(or equivalent) Implied by four of B, D, F, H, J in the same set* Give A1 if one element is in the wrong set; or if two elements are interchanged
(v)	P and M are not isomorphic M is abelian, P is non-abelian	B1 B1 **2**	Valid reason e.g. M has one element of order 2 P has more than one
(vi)	(see table below)	B3 **3**	Give B2 for 7 correct B1 for 4 correct
(vii)	$\{E, B\}, \{E, D\}, \{E, F\}, \{E, H\}, \{E, J\}$ $\{E, A, C, G, I\}$	M1 A1 ft B2 cao **4**	Ignore $\{E\}$ and P *Subgroups of order 2* Using elements of order 2 (allow two errors/omissions) Correct or ft. A0 if any others given *Subgroups of order greater than 2* Deduct 1 mark (from B2) for each extra subgroup given

(vi)

	A	B	C	D	E	F	G	H	I	J
Order	5	2	5	2	1	2	5	2	5	2

MEI FP3 2008

4 (i)	*Commutative:* $x*y=y*x$ (for all *x*, *y*) *Associative:* $(x*y)*z=x*(y*z)$ (for all *x*, *y*, *z*)	B1 B2 **3**	Accept e.g. 'Order does not matter' Give B1 for a partial explanation, e.g. 'Position of brackets does not matter'
(ii)	$2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=2xy+x+y+\frac{1}{2}-\frac{1}{2}$ $=2xy+x+y=x*y$	B1 ag **1**	*Intermediate step required*
(iii)(*A*)	If $x,y\in S$ then $x>-\frac{1}{2}$ and $y>-\frac{1}{2}$ $x+\frac{1}{2}>0$ and $y+\frac{1}{2}>0$, so $2(x+\frac{1}{2})(y+\frac{1}{2})>0$ $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}>-\frac{1}{2}$, so $x*y\in S$	M1 A1 A1 **3**	
(*B*)	0 is the identity since $0*x=0+x+0=x$ If $x\in S$ and $x*y=0$ then $2xy+x+y=0$ $y=\frac{-x}{2x+1}$ $y+\frac{1}{2}=\frac{1}{2(2x+1)}>0$ (since $x>-\frac{1}{2}$) so $y\in S$ *S* is closed and associative; there is an identity; and every element of *S* has an inverse in *S*	B1 B1 M1 A1 M1 A1 **6**	or $2(x+\frac{1}{2})(y+\frac{1}{2})-\frac{1}{2}=0$ or $y+\frac{1}{2}=\frac{1}{4(x+\frac{1}{2})}$ *Dependent on M1A1M1*
(iv)	If $x*x=0$, $2x^2+x+x=0$ $x=0$ or -1 0 is the identity (and has order 1) -1 is not in *S*	M1 A1 A1 **3**	
(v)	$4*6=48+4+6=58$ $=56+2=7\times8+2$ So $4\circ6=2$	B1 B1 ag **2**	
(vi)	Element: 0, 1, 2, 4, 5, 6 Order: 1, 6, 6, 3, 3, 2	B3 **3**	Give B2 for 4 correct B1 for 2 correct
(vii)	$\{0\}$, G $\{0,\ 6\}$ $\{0,\ 4,\ 5\}$	B1 B1 B1 **3**	*Condone omission of G* If more than 2 non-trivial subgroups are given, deduct 1 mark (from final B1B1) for each non-trivial subgroup in excess of 2

Element	0	1	2	4	5	6
Order	1	6	6	3	3	2

MEI FP3 2009

4 (i)	In *G*, $3^2=2,\ 3^3=6,\ 3^4=4,\ 3^5=5,\ 3^6=1$ [or $5^2=4,\ 5^3=6,\ 5^4=2,\ 5^5=3,\ 5^6=1$] In *H*, $5^2=7,\ 5^3=17,\ 5^4=13,\ 5^5=11,\ 5^6=1$ [or $11^2=13,\ 11^3=17,\ 11^4=7,\ 11^5=5,\ 11^6=1$] *G* has an element 3 (or 5) of order 6 *H* has an element 5 (or 11) of order 6	M1 A1 B1 B1 **4**	All powers of an element of order 6 All powers correct in both groups
(ii)	$\{1,\ 6\}$ $\{1,\ 2,\ 4\}$	B1 B2 **3**	*Ignore* $\{1\}$ *and G* *Deduct 1 mark (from B1B2) for each proper subgroup in excess of two*
(iii)	*G* *H*: $1\leftrightarrow1$, $2\leftrightarrow7$, $3\leftrightarrow5$, $4\leftrightarrow13$, $5\leftrightarrow11$, $6\leftrightarrow17$ OR *G* *H*: $1\leftrightarrow1$, $2\leftrightarrow13$, $3\leftrightarrow11$, $4\leftrightarrow7$, $5\leftrightarrow5$, $6\leftrightarrow17$	B4 **4**	Give B3 for 4 correct, B2 for 3 correct, B1 for 2 correct
(iv)	$\mathrm{ad}(1)=\mathrm{a}(3)=1$ $\mathrm{ad}(2)=\mathrm{a}(2)=3$ $\mathrm{ad}(3)=\mathrm{a}(1)=2$, so $\mathrm{ad}=\mathrm{c}$ $\mathrm{da}(1)=\mathrm{d}(2)=2$ $\mathrm{da}(2)=\mathrm{d}(3)=1$ $\mathrm{da}(3)=\mathrm{d}(1)=3$, so $\mathrm{da}=\mathrm{f}$	M1 A1 M1 A1 **4**	Evaluating e.g. $\mathrm{ad}(1)$ *(one case sufficient; intermediate value must be shown)* For $\mathrm{ad}=\mathrm{c}$ correctly shown Evaluating e.g. $\mathrm{da}(1)$ *(one case sufficient; no need for any working)*
(v)	*S* is not abelian; *G* is abelian	B1 **1**	*or S* has 3 elements of order 2; *G* has 1 element of order 2 *or S* is not cyclic *etc*
(vi)	Element: a, b, c, d, e, f Order: 3, 3, 2, 2, 1, 2	B4 **4**	Give B3 for 5 correct, B2 for 3 correct, B1 for 1 correct
(vii)	$\{e,\ c\}$, $\{e,\ d\}$, $\{e,\ f\}$ $\{e,\ a,\ b\}$	B1B1B1 B1 **4**	Ignore { e } and *S* If more than 4 proper subgroups are given, deduct 1 mark for each proper subgroup in excess of 4

MEI FP3 2010

<table>
<tr><td>4 (i)</td><td>$\mathrm{s\,t}(x)=\mathrm{s}\left(\dfrac{x}{x-1}\right)=\dfrac{\dfrac{x}{x-1}-1}{\dfrac{x}{x-1}}$
$=\dfrac{x-(x-1)}{x}=\dfrac{1}{x}=\mathrm{r}(x)$
$\mathrm{t\,s}(x)=\mathrm{t}\left(\dfrac{x-1}{x}\right)=\dfrac{\dfrac{x-1}{x}}{\dfrac{x-1}{x}-1}$
$=\dfrac{x-1}{(x-1)-x}=1-x=\mathrm{q}(x)$</td><td>M1
A1 (ag)
M1
A1
4</td><td></td></tr>
<tr><td>(ii)</td><td><table><tr><td></td><td>p</td><td>q</td><td>r</td><td>s</td><td>t</td><td>u</td></tr><tr><td>p</td><td>p</td><td>q</td><td>r</td><td>s</td><td>t</td><td>u</td></tr><tr><td>q</td><td>q</td><td>p</td><td>s</td><td>r</td><td>u</td><td>t</td></tr><tr><td>r</td><td>r</td><td>u</td><td>p</td><td>t</td><td>s</td><td>q</td></tr><tr><td>s</td><td>s</td><td>t</td><td>q</td><td>u</td><td>r</td><td>p</td></tr><tr><td>t</td><td>t</td><td>s</td><td>u</td><td>q</td><td>p</td><td>r</td></tr><tr><td>u</td><td>u</td><td>r</td><td>t</td><td>p</td><td>q</td><td>s</td></tr></table></td><td>B3
3</td><td>Give B2 for 4 correct, B1 for 2 correct</td></tr>
<tr><td>(iii)</td><td><table><tr><td>Element</td><td>p</td><td>q</td><td>r</td><td>s</td><td>t</td><td>u</td></tr><tr><td>Inverse</td><td>p</td><td>q</td><td>r</td><td>u</td><td>t</td><td>s</td></tr></table></td><td>B3
3</td><td>Give B2 for 4 correct, B1 for 2 correct</td></tr>
<tr><td>(iv)</td><td>{ p }, F
{ p, q }, { p, r }, { p, t }
{ p, s, u }</td><td>B1B1B1
B1
4</td><td>Ignore these in the marking
Deduct one mark for each non-trivial subgroup in excess of four</td></tr>
<tr><td>(v)</td><td><table><tr><td>Element</td><td>1</td><td>-1</td><td>$e^{\frac{\pi}{3}j}$</td><td>$e^{-\frac{\pi}{3}j}$</td><td>$e^{\frac{2\pi}{3}j}$</td><td>$e^{-\frac{2\pi}{3}j}$</td></tr><tr><td>Order</td><td>1</td><td>2</td><td>6</td><td>6</td><td>3</td><td>3</td></tr></table></td><td>B4
4</td><td>Give B3 for 4 correct, B2 for 3 correct
B1 for 2 correct</td></tr>
<tr><td>(vi)</td><td>$2^1=2,\ 2^2=4,\ 2^3=8,\ 2^4=16,\ 2^5=13,\ 2^6=\mathbf{7}$
$2^7=14,\ 2^8=9,\ 2^9=\mathbf{18},\ 2^{10}=17,\ 2^{11}=15,\ 2^{12}=11$
$2^{13}=3,\ 2^{14}=6,\ 2^{15}=12,\ 2^{16}=5,\ 2^{17}=10,\ 2^{18}=1$
Hence 2 has order 18</td><td>M1
A1
A1
3</td><td>Finding (at least two) powers of 2
For $2^6=7$ and $2^9=18$
Correctly shown
All powers listed implies final A1</td></tr>
<tr><td>(vii)</td><td>G is abelian (so all its subgroups are abelian)
F is not abelian</td><td>B1
1</td><td>Can have 'cyclic' instead of 'abelian'</td></tr>
<tr><td>(viii)</td><td>Subgroup of order 6 is $\{1,\ 2^3,\ 2^6,\ 2^9,\ 2^{12},\ 2^{15}\}$
i.e. $\{1,\ 7,\ 8,\ 11,\ 12,\ 18\}$</td><td>M1
A1
2</td><td>or B2</td></tr>
</table>

MEI FP3 2011

<table>
<tr><td>
<table>
<tr><td></td><td>1</td><td>3</td><td>4</td><td>5</td><td>9</td></tr>
<tr><td>1</td><td>1</td><td>3</td><td>4</td><td>5</td><td>9</td></tr>
<tr><td>3</td><td>3</td><td>9</td><td>1</td><td>4</td><td>5</td></tr>
<tr><td>4</td><td>4</td><td>1</td><td>5</td><td>9</td><td>3</td></tr>
<tr><td>5</td><td>5</td><td>4</td><td>9</td><td>3</td><td>1</td></tr>
<tr><td>9</td><td>9</td><td>5</td><td>3</td><td>1</td><td>4</td></tr>
</table>
Composition table shows closure
Identity is 1
<table>
<tr><td>Element</td><td>1</td><td>3</td><td>4</td><td>5</td><td>9</td></tr>
<tr><td>Inverse</td><td>1</td><td>4</td><td>3</td><td>9</td><td>5</td></tr>
</table>
So every element has an inverse
</td><td>B2

B1
B1

B2
6</td><td>Give B1 if not more than 4 errors

Dependent on B2 for table

Give B1 for 3 correct</td></tr>
<tr><td>Since 5 is prime,
a group of order 5 must be cyclic
Two cyclic groups of the same order must be isomorphic</td><td>B1
B1
B1
3</td><td></td></tr>
<tr><td>
<table>
<tr><td>H</td><td>1</td><td>$e^{\frac{2}{5}\pi j}$</td><td>$e^{\frac{4}{5}\pi j}$</td><td>$e^{\frac{6}{5}\pi j}$</td><td>$e^{\frac{8}{5}\pi j}$</td></tr>
<tr><td>G</td><td>1</td><td>3</td><td>9</td><td>5</td><td>4</td></tr>
<tr><td>or</td><td>1</td><td>4</td><td>5</td><td>9</td><td>3</td></tr>
<tr><td>or</td><td>1</td><td>5</td><td>3</td><td>4</td><td>9</td></tr>
<tr><td>or</td><td>1</td><td>9</td><td>4</td><td>3</td><td>5</td></tr>
</table>
</td><td>B1
B2
3</td><td>For $1 \leftrightarrow 1$
For non-identity elements</td></tr>
<tr><td>Identity is $(1,\ 1)$
Inverse of $(9,\ 3)$ is $(5,\ 4)$</td><td>B1
B1
2</td><td></td></tr>
<tr><td>$(x,\ y)^5 = (x^5,\ y^5)$
Since G has order 5, $x^5 = 1$ and $y^5 = 1$
Hence $(x,\ y)^5 = (1,\ 1)$</td><td>M1
M1
A1 (ag)
3</td><td></td></tr>
<tr><td>Order of $(x,\ y)$ is a factor of 5 (so must be 1 or 5)
Only identity $(1,\ 1)$ can have order 1
Hence all other elements have order 5</td><td>M1
B1
A1 (ag)
3</td><td></td></tr>
<tr><td>$\{(1,\ 1),\ (9,\ 3),\ (4,\ 9),\ (3,\ 5),\ (5,\ 4)\}$</td><td>B2
2</td><td>Give B1 ft for 5 elements including $(1,\ 1),\ (9,\ 3),\ (5,\ 4)$</td></tr>
<tr><td>An element of order 5 generates a subgroup, and so can be in only one subgroup of order 5

Number is $24 \div 4 = 6$</td><td>M1

A1
2</td><td>Or for $24 \div 4$
Or listing at least 2 other subgroups
Give B1 for unsupported answer 6</td></tr>
</table>

MEI FP3 2012

(i)	$\begin{array}{c\|cccc} P & 1 & 5 & 7 & 11 \\ \hline 1 & 1 & 5 & 7 & 11 \\ 5 & 5 & 1 & 11 & 7 \\ 7 & 7 & 11 & 1 & 5 \\ 11 & 11 & 7 & 5 & 1 \end{array}$ Table shows closure Identity is 1 All elements are self-inverse	B1 B1 B1 B1 **[4]**	 *Condone no mention of inverse of 1*	
(ii)	$(xy)(y^{-1}x^{-1})$ $= x(yy^{-1})x^{-1} = xex^{-1} = xx^{-1} = e$ So $y^{-1}x^{-1}$ is the inverse of xy	M1 E1 **[2]**	Or $(y^{-1}x^{-1})(xy)$	
(iii)	$a^{-1} = a,\ b^{-1} = b,\ c^{-1} = c,\ c^{-1} = (ab)^{-1} = b^{-1}a^{-1}$ Hence $c = ba$	M1 E1 **[2]**	For any one of these	
(iv)	$bc = b(ba)$ $bc = ea = a$ $ac = a(ab) = eb = b$ $cb = a,\ \ ca = b$	M1 E1 E1 B1 **[4]**	Or $ba = c \Rightarrow a = b^{-1}c$	Any correct first step
(v)	$\begin{array}{c\|cccc} R & e & a & b & c \\ \hline e & e & a & b & c \\ a & a & e & c & b \\ b & b & c & e & a \\ c & c & b & a & e \end{array}$ R is closed Hence R is a subgroup Same pattern as P; hence R and P are isomorphic	B1 M1 E1 E1 **[4]**	 No need to mention identity or inverses *Dependent on B1 (only)*	
(vi)	$\begin{array}{c\|cccccccc} \text{Element} & A & B & C & D & E & F & G & H \\ \hline \text{Order} & 2 & 2 & 2 & 2 & 1 & 4 & 2 & 4 \end{array}$	B3 **[3]**	Give B1 for 3 correct; B2 for 6 correct	
(vii)	$\{E, A\}, \{E, B\}, \{E, C\}, \{E, D\}, \{E, G\}$ $\{E, F, G, H\}$ $\{E, A, B, G\}$ $\{E, C, D, G\}$	B2 B1 B1 B1 **[5]**	Ignore $\{E\}$ and T in the marking Give B1 for 3 correct	Deduct 1 mark (from this B2) for each subgroup of order 2 given in excess of five Deduct 1 mark (from this B1B1B1) for each subgroup of order 3 or more given in excess of three

MEI FP3 2013

(a)	**(i)**	Identity is e Element: a b c d e f g h Inverse: b a c g e h d f	B1 B2 **[3]**	Give B1 for four correct	
(a)	**(ii)**	$d^2 = a,\ \ d^4 = c$ Hence d has order 8, and G is cyclic	M1 A1 A1 E1 **[4]**	Finding powers of an element Identifying d (or f or g or h) as a generator Or $f^2 = b,\ \ f^4 = c$ Or $g^2 = b,\ \ g^4 = c$ Or $h^2 = a,\ \ h^4 = c$ Correctly shown	At least fourth power *Implies previous M1*
(a)	**(iii)**	H: 0, 2, 4, 6, 8, 10, 12, 14 G: e d a f c h b g or: e f b d c g a h or: e g b h c f a d or: e h a g c d b f	B1 B1 B1 **[3]**	For $e \leftrightarrow 0$ and $c \leftrightarrow 8$ For $\{d, f, g, h\} \leftrightarrow \{2, 6, 10, 14\}$ For a fully correct isomorphism	In any order
(a)	**(iv)**	Rotations have order 2 or 4 Reflections have order 2 There is no element of order 8 Hence not isomorphic	B1 E1 **[2]**	Correct statement about rotations and/or reflections which implies non-IM *Or* More than one element of order 2 *Or* Not commutative Fully correct explanation	*Or* (4) reflections (and 180° rotation) have order 2 *Or* composition of reflections (or 90° rotation and reflection) is not commutative Dependent on previous B1

(b)	**(i)**	$\mathrm{f}_m\mathrm{f}_n(x) = \dfrac{\dfrac{x}{1+nx}}{1+m\left(\dfrac{x}{1+nx}\right)}$ $= \dfrac{x}{1+nx+mx} = \dfrac{x}{1+(m+n)x} = \mathrm{f}_{m+n}(x)$	M1 E1 **[2]**	Composition of functions Correctly shown	In either order E0 if in wrong order
(b)	**(ii)**	$(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n}\mathrm{f}_p = \mathrm{f}_{m+n+p}$ $\mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p) = \mathrm{f}_m\mathrm{f}_{n+p} = \mathrm{f}_{m+n+p}$ Hence S is associative	M1 E1 **[2]**	Combining three functions Correctly shown	M1E1 bod for $(\mathrm{f}_m\mathrm{f}_n)\mathrm{f}_p = \mathrm{f}_{m+n+p} = \mathrm{f}_m(\mathrm{f}_n\mathrm{f}_p)$
(b)	**(iii)**	For any f_m, f_n in S, $\mathrm{f}_m\mathrm{f}_n = \mathrm{f}_{m+n}$ $\mathrm{f}_m\mathrm{f}_n$ is in S (so S is closed) Identity is f_0 Inverse of f_n is f_{-n} since $\mathrm{f}_n\mathrm{f}_{-n} = \mathrm{f}_{n-n} = \mathrm{f}_0$ S is also associative, and hence is a group	M1 A1 B1 B1 B1 E1 **[6]**	Referring to this in context B0 for x B1 for $n = 0$ Closure, associativity, identity and inverses must all be mentioned in (iii)	Dependent on previous 5 marks
(b)	**(iv)**	$\{\mathrm{f}_{2n}\}$ for all integers n	B2 **[2]**	Or $\{\mathrm{f}_{3n}\}$, etc Give B1 for multiples of 2 (or 3, etc) but not completely correctly described	e.g. $\{\mathrm{f}_0, \mathrm{f}_2, \mathrm{f}_4, \mathrm{f}_6, \ldots\}$

MEI FP3 2014

(i)		$(a^2b)^2 = a^4b^2 = a^4$ $(a^2b)^3 = b,\ (a^2b)^4 = a^2,\ (a^2b)^5 = a^4b$ $(a^2b)^6 = e$ Hence a^2b has order 6	M1 A1 E1 **[3]**	Finding one power Three powers correct Fully correct explanation	*No need to state conclusion, provided it has been fully justified*
(ii)		$\begin{array}{c\|cccc} & e & a^3 & b & a^3b \\ \hline e & e & a^3 & b & a^3b \\ a^3 & a^3 & e & a^3b & b \\ b & b & a^3b & e & a^3 \\ a^3b & a^3b & b & a^3 & e \end{array}$ The set is closed; hence it is a subgroup of G	B2 B1 **[3]**	Give B1 for no more than three errors or omissions *'Closed' (or equivalent) is required*	
(iii)		$\{e,\ a^3\},\ \{e,\ b\},\ \{e,\ a^3b\}$ $\{e,\ a^2,\ a^4\}$ $\{e,\ a,\ a^2,\ a^3,\ a^4,\ a^5\}$ $\{e,\ a^2b,\ a^4,\ b,\ a^2,\ a^4b\}$ $\{e,\ ab,\ a^2,\ a^3b,\ a^4,\ a^5b\}$	B2 B1 B1 B1 B1 **[6]**	Give B1 for one correct B0 if any other set of order 3 *No mark for $\{e\}$. Deduct one mark (out of B6) for each set (including G) of order other than 1, 2, 3, 6*	*Deduct one mark (out of B2) for each set of order 2 in excess of 3* *Deduct one mark (out of B3) for each set of order 6 in excess of 3*
(iv)		$11^2 = 31,\ 11^3 = 71,\ 11^4 = 61,\ 11^5 = 41,\ 11^6 = 1$ $17^2 = 19,\ 17^3 = 53,\ 17^4 = 1$ 11 has order 6 17 has order 4 $19^2 = 1$; 19 has order 2	M1 A1 A1 B1 **[4]**	Finding at least two powers of 11 (or 17) *Either correct implies M1*	
(v)		$\{1,\ 17,\ 19,\ 53\}$	M1 A1 **[2]**	Selecting powers of 17 Or B2 for $\{1,\ 37,\ 19,\ 73\}$	
(vi)	(*A*)	Taking $a = 11,\ b = 19$ $1, 11, 11^2, \ldots, 11^5, 19, 11\times19, 11^2\times19, \ldots, 11^5\times19$ $\{1, 11, 31, 71, 61, 41, 19, 29, 49, 89, 79, 59\}$ i.e. $\{1, 11, 19, 29, 31, 41, 49, 59, 61, 71, 79, 89\}$	B1 M1 A1 **[3]**	*There are (many) other possibilities* Finding elements of G using their a, b	
(vi)	(*B*)	$1,\ 11^3,\ 19,\ 11^3\times19$ $\{1,\ 71,\ 19,\ 89\}$	M1 M1 A1 **[3]**	Reference to group in (ii) Finding group in (ii) with their a, b	

MEI FP3 2016

(a)	(i)	3*(9*11) = 3*3 = 9 (3*9)*11 = 11*11 = 9 Construction of group table (or otherwise): It shows closure, the identity is 1 each element has an inverse $3^{-1}=11,\ 9^{-1}=9, 11^{-1}=3, 1^{-1}=1$	B1 B1 B1 B1 B1 B1		Group table is: $\begin{array}{c\|cccc} & 1 & 3 & 9 & 11 \\ \hline 1 & 1 & 3 & 9 & 11 \\ 3 & 3 & 9 & 11 & 1 \\ 9 & 9 & 11 & 1 & 3 \\ 11 & 11 & 1 & 3 & 9 \end{array}$
			[6]		
	(ii)	Element: 1, 3, 9, 11 Order: 1, 4, 2, 4	B2	−1 each error	
			[2]		
	(iii)	{1} {1,9} {1,3,9,11}	B1	*Condone omission of trivial subgroups*	*B0 if any extras*
			[1]		
	(iv)	e.g. $3^2 = 9,\ 3^3 = 11, 3^4 = 1$ 3 generates the group and so it is cyclic	E1		
			[1]		
(b)		Composition table: $\begin{array}{c\|cccc} & e & a & b & ab \\ \hline e & e & a & b & ab \\ a & a & e & ab & b \\ b & b & ab & e & a \\ ab & ab & b & a & e \end{array}$ All elements are self-inverse, and so no element generates the group	B3 E1	−1 each error	
			[4]		
(c)		In group G all elements are self-inverse i.e. $X^2 = I$, $Y^2 = I$ and $Z^2 = I$ So this group is isomorphic to the group in (b) e.g. $\mathbf{I} \leftrightarrow e\quad \mathbf{X} \leftrightarrow a\quad \mathbf{Y} \leftrightarrow b\quad \mathbf{Z} \leftrightarrow ab$	M1 A1A1 A1 B1B1	Correctly shown	
			[6]		
(d)		One of the elements needs to be the identity element. It is neither p nor q for otherwise p ° q = p (or q) It is neither r nor s, for otherwise p ° q = q ° p = r (or s) So there is no identity element and so not a group	M1 A1 A1 E1		
			[4]		