"The Knowledge" FOR EXTRA PURE

The form of the complementary function given the roots of the auxiliary equation for a second order recurrence relation

Roots of auxiliary equation	Complementary function
Real and distinct λ_1 and λ_2	$p \times \lambda_1^n + q \times \lambda_2^n$
Repeated real root λ	$(p + qn)\lambda^n$
Complex roots $r(\cos\theta + i\sin\theta)$	$r^n(p\cos(n\theta) + q\sin(n\theta))$

How to change your trial function for a recurrence relation if the natural choice appears in the complementary function

Multiply the trial function by n

The equation satisfied by an eigenvector and its corresponding eigenvalue

$\boldsymbol{Mv} = \lambda\boldsymbol{v}$ or $(\boldsymbol{M} - \lambda\boldsymbol{I})\boldsymbol{v} = \boldsymbol{0}$

The eigenvalues for a matrix representing a reflection, and the geometrical significance of its eigenvectors

Eigenvalue of 1 corresponding to an eigenvector that is the mirror line (or in the plane of reflection in 3D).

Eigenvalue of -1 corresponding to an eigenvector that is perpendicular to the mirror line or plane of reflection.

The eigenvalues for a matrix representing a rotation, and the geometrical significance of its eigenvectors

In 2D: no eigenvalues.

In 3D: eigenvalue of 1 corresponding to an eigenvector that is the axis of rotation.

The result for diagonalising a matrix

$\boldsymbol{M} = \boldsymbol{PDP}^{-1}$ (or sometimes written as $\boldsymbol{D} = \boldsymbol{P}^{-1}\boldsymbol{MP}$), where $\boldsymbol{D}$ has diagonal entries that are the eigenvalues of $\boldsymbol{M}$ and $\boldsymbol{P}$ has columns that are the eigenvectors of $\boldsymbol{M}$

The definition of the characteristic equation

The equation $\det(\boldsymbol{M} - \lambda\boldsymbol{I}) = 0$

The Cayley-Hamilton theorem

A matrix satisfies its own characteristic equation.

The criteria for $(G,*)$ to be a group

G is closed under $*$, $*$ is associative, G contains an identity element, and every element in G has an inverse that is also in G.

The definition of an Abelian group

A group where the operation is commutative.

The definition of a cyclic group

A group with an element that has the same order as the order of the group.

Lagrange's theorem

If H is a subgroup of a finite group G, then the order of H divides the order of G.

The meaning of a section and of a contour of a surface

A section has a fixed *x* or a fixed y value. A contour has a fixed z value.

The meaning and significance of **grad** g

$$\nabla \mathrm{g} = \mathbf{grad}\ \mathrm{g} = \begin{pmatrix} \frac{\partial \mathrm{g}}{\partial x} \\ \frac{\partial \mathrm{g}}{\partial y} \\ \frac{\partial \mathrm{g}}{\partial z} \end{pmatrix}. \text{ If } \mathrm{g}(x, y, z) \text{ can be written as } z = \mathrm{f}(x, y) \text{ then } \mathbf{grad}\ \mathrm{g} = \begin{pmatrix} \frac{\partial \mathrm{f}}{\partial x} \\ \frac{\partial \mathrm{f}}{\partial y} \\ -1 \end{pmatrix}$$

(in the formula booklet).
grad g is perpendicular to the surface.