Matrices 2 Past Paper Question Pack

MEI P6 June 02

You are given the matrix $\mathbf{M} = \begin{pmatrix} k & 3 \\ 0 & 2 \end{pmatrix}$, where $k \neq 2$.

(i) Find the eigenvalues of $\mathbf{M}$, and the corresponding eigenvectors. [7]

(ii) Write down a matrix $\mathbf{P}$ for which $\mathbf{P}^{-1}\mathbf{M}\mathbf{P}$ is a diagonal matrix. [2]

(iii) Hence find the matrix $\mathbf{M}^n$. [7]

(iv) For the case $k = 1$, use the Cayley-Hamilton theorem to find integers p and q such that

$$\mathbf{M}^9 = p\mathbf{M}^8 + q\mathbf{M}^7.$$ [4]

MEI P6 Jun 04

(a) **(i)** Find the inverse of the matrix $\begin{pmatrix} 3 & -7 & k \\ 2 & 2 & 5 \\ 1 & 3 & 4 \end{pmatrix}$ where $k \neq 0$. [6]

The matrix $\mathbf{A}$ has eigenvectors $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} -7 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix}$ with corresponding eigenvalues -1, 1, 0 respectively.

(ii) Express $\mathbf{A}$ as the product of three matrices, and hence find $\mathbf{A}$. [5]

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The matrix $\mathbf{M}$ is $\begin{pmatrix} 2 & 6 & 8 \\ 2 & 3 & 5 \\ -6 & 8 & 2 \end{pmatrix}$.

(i) Verify that $\begin{pmatrix} 19 \\ 13 \\ -5 \end{pmatrix}$ is an eigenvector of $\mathbf{M}$, and find the corresponding eigenvalue. [4]

(ii) Find the other two eigenvalues of $\mathbf{M}$, and find corresponding eigenvectors. [8]

(iii) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{M}^4\mathbf{P} = \mathbf{D}$. [3]

(iv) Use the Cayley-Hamilton theorem to show that $\mathbf{M}^3 = 7\mathbf{M}^2 - 12\mathbf{M}$. [2]

(v) Find a and b such that $\mathbf{M}^4 = a\mathbf{M}^2 + b\mathbf{M}$. [3]

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(i) Find the eigenvalues and eigenvectors of the matrix **M**, where

$$\mathbf{M} = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}.$$

Hence express **M** in the form $\mathbf{PDP}^{-1}$ where **D** is a diagonal matrix. **[8]**

(ii) Write down an equation for $\mathbf{M}^n$ in terms of the matrices **P** and **D**.

Hence obtain expressions for the elements of $\mathbf{M}^n$.

Show that $\mathbf{M}^n$ tends to a limit as n tends to infinity. Find that limit. **[6]**

(iii) Express $\mathbf{M}^{-1}$ in terms of the matrices **P** and **D**. Hence determine whether or not $(\mathbf{M}^{-1})^n$ tends to a limit as n tends to infinity. **[4]**

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The matrix $\mathbf{Q}$ is given by $\mathbf{Q} = \begin{pmatrix} 3 & 3 \\ 4 & 7 \end{pmatrix}$.

(i) Find the eigenvalues and corresponding eigenvectors of $\mathbf{Q}$. **[5]**

(ii) State a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{Q} = \mathbf{PDP}^{-1}$. **[2]**

(iii) Show that, for $n \geqslant 1$, $\mathbf{Q}^n = \frac{1}{8}\begin{pmatrix} 6+2\varphi & 3\varphi-3 \\ 4\varphi-4 & 6\varphi+2 \end{pmatrix}$, where $\varphi = 9^n$. **[4]**

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(i) Find the inverse of the matrix $\begin{pmatrix} 1 & 3 & 2 \\ -1 & 2 & k \\ k & 1 & 6 \end{pmatrix}$. **[5]**

The matrix **M** has eigenvalues 1, 2 and 3. The corresponding eigenvectors are $\begin{pmatrix} 2 \\ 0 \\ 6 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ respectively.

(ii) Write down the matrix **P** such that $\mathbf{M} = \mathbf{PDP}^{-1}$ where $\mathbf{D} = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. **[2]**

(iii) Hence find **M**. **[5]**

(iv) Find constants a, b and c such that $\mathbf{M}^{-1} = a\mathbf{M}^2 + b\mathbf{M} + c\mathbf{I}$. **[6]**