Matrices Past Paper Question Pack – Mark Schemes

MEI FP2 Jan 2006

3 (i)	$(1-\lambda)[\,(-3-\lambda)(-4-\lambda)-12\,]$ $-2[\,-2(-4-\lambda)-12\,]+3[\,-4-2(-3-\lambda)\,]=0$ $(1-\lambda)(\lambda^2+7\lambda)-2(2\lambda-4)+3(2\lambda+2)=0$ $\lambda^3+6\lambda^2-9\lambda-14=0$	M1 A1 A1 (ag) **3**	Evaluating $\det(\mathbf{M}-\lambda\mathbf{I})$ *Allow one omission and two sign errors* $\det(\mathbf{M}-\lambda\mathbf{I})$ correct Correctly obtained (=0 is required)
(ii)	When $\lambda=-1$, $-1+6+9-14=0$ $(\lambda+1)(\lambda^2+5\lambda-14)=0$ $(\lambda+1)(\lambda-2)(\lambda+7)=0$ Other eigenvalues are $2,\ -7$	B1 M1 A1 **3**	or showing that $(\lambda+1)$ is a factor, and deducing that -1 is a root for $(\lambda+1)\times$ quadratic factor
(iii)	$x+2y+3z=-x$ $-2x-3y+6z=-y$ $2x+2y-4z=-z$ $z=0,\ x+y=0$ An eigenvector is $\begin{pmatrix}1\\-1\\0\end{pmatrix}$	M1 M1 A1 **3**	At least two equations Solving to obtain an eigenvector
	OR $\begin{pmatrix}2\\2\\3\end{pmatrix}\times\begin{pmatrix}-2\\-2\\6\end{pmatrix}=\begin{pmatrix}18\\-18\\0\end{pmatrix}$ M1 M1 A1		Appropriate vector product Evaluation of vector product
(iv)	$\mathbf{M}\begin{pmatrix}3\\0\\1\end{pmatrix}=\begin{pmatrix}6\\0\\2\end{pmatrix}=2\begin{pmatrix}3\\0\\1\end{pmatrix}$ $\mathbf{M}\begin{pmatrix}0\\3\\-2\end{pmatrix}=\begin{pmatrix}0\\-21\\14\end{pmatrix}=-7\begin{pmatrix}0\\3\\-2\end{pmatrix}$	M1 A1A1 **3**	Any method for verifying or finding an eigenvector
(v)	$\mathbf{P}=\begin{pmatrix}1&3&0\\-1&0&3\\0&1&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}-1&0&0\\0&2&0\\0&0&-7\end{pmatrix}^3$ $=\begin{pmatrix}-1&0&0\\0&8&0\\0&0&-343\end{pmatrix}$	B1 ft M1 A1 ft **3**	seen or implied (ft) *(condone eigenvalues in wrong order)* Order must be consistent with **P** (when B1 has been awarded)
(vi)	By CHT, $\mathbf{M}^3+6\mathbf{M}^2-9\mathbf{M}-14\mathbf{I}=\mathbf{0}$ $\mathbf{M}^2+6\mathbf{M}-9\mathbf{I}-14\mathbf{M}^{-1}=\mathbf{0}$ $\mathbf{M}^{-1}=\frac{1}{14}\mathbf{M}^2+\frac{3}{7}\mathbf{M}-\frac{9}{14}\mathbf{I}$	B1 M1 A1	Condone omission of **I** Condone dividing by **M**

MEI FP2 Jan 2007

3 (i)	$\det \mathbf{P} = 1(6-k) - 1(4-2)$ $= 4-k$ $\mathbf{P}^{-1} = \frac{1}{4-k}\begin{pmatrix} -1 & 2 & 6-k \\ 4 & -4-k & k-12 \\ -1 & 2 & 2 \end{pmatrix}$ When $k=2$, $\mathbf{P}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & 2 & 4 \\ 4 & -6 & -10 \\ -1 & 2 & 2 \end{pmatrix}$	M1 A1 M1 M1 A1 ft B1 ag **6**	Evaluating at least three cofactors Fully correct method for inverse Ft from wrong determinant Correctly obtained
(ii)	$\mathbf{M}\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = 0\begin{pmatrix} 4 \\ 1 \\ 1 \end{pmatrix}$ $\quad \mathbf{M}\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} = 1\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ $\mathbf{M}\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix} = 2\begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$ Eigenvalues are 0, 1, 2	M1 A1A1A1 **4**	For one evaluation
	OR M1 Eigenvalues are 0, 1, 2 A2 A1		Obtaining an eigenvalue (e.g. by solving $-\lambda^3 + 3\lambda^2 - 2\lambda = 0$) Give A1 for one correct Verifying given eigenvectors, linking with eigenvalues correctly
(iii)	$\mathbf{M}^n = \begin{pmatrix} 4 & 2 & 2 \\ 1 & 1 & 3 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix}\frac{1}{2}\begin{pmatrix} -1 & 2 & 4 \\ 4 & -6 & -10 \\ -1 & 2 & 2 \end{pmatrix}$ $= \frac{1}{2}\begin{pmatrix} 0 & 2 & 2^{n+1} \\ 0 & 1 & 3\times 2^n \\ 0 & 0 & -2^n \end{pmatrix}\begin{pmatrix} -1 & 2 & 4 \\ 4 & -6 & -10 \\ -1 & 2 & 2 \end{pmatrix}$ $= \begin{pmatrix} 4-2^n & -6+2^{n+1} & -10+2^{n+1} \\ 2-3\times 2^{n-1} & -3+3\times 2^n & -5+3\times 2^n \\ 2^{n-1} & -2^n & -2^n \end{pmatrix}$ $= \begin{pmatrix} 4 & -6 & -10 \\ 2 & -3 & -5 \\ 0 & 0 & 0 \end{pmatrix} + 2^{n-1}\begin{pmatrix} -2 & 4 & 4 \\ -3 & 6 & 6 \\ 1 & -2 & -2 \end{pmatrix}$	B1B1 M1A1 B1 ft M1 A1 A1 ag **8**	For $\begin{pmatrix} 4 & 2 & 2 \\ 1 & 1 & 3 \\ 1 & 0 & -1 \end{pmatrix}$ and $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2^n \end{pmatrix}$ seen (for B2, these must be consistent) For $\mathbf{S}\mathbf{D}^n\mathbf{S}^{-1}$ (M1A0 if order wrong) *or* $\frac{1}{2}\begin{pmatrix} 4 & 2 & 2 \\ 1 & 1 & 3 \\ 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 & 0 \\ 4 & -6 & -10 \\ -2^n & 2^{n+1} & 2^{n+1} \end{pmatrix}$ Evaluating product of 3 matrices Any correct form

3 (i)	$\det(\mathbf{M}-\lambda\mathbf{I})=(3-\lambda)[(3-\lambda)(-4-\lambda)-4]$ $-5[5(-4-\lambda)+4]+2[-10-2(3-\lambda)]$ $=(3-\lambda)(-16+\lambda+\lambda^2)-5(-16-5\lambda)+2(-16+2\lambda)$ $=-48+19\lambda+2\lambda^2-\lambda^3+80+25\lambda-32+4\lambda$ $=48\lambda+2\lambda^2-\lambda^3$ Characteristic equation is $\lambda^3-2\lambda^2-48\lambda=0$	M1 A1 M1 A1 ag **4**	Obtaining $\det(\mathbf{M}-\lambda\mathbf{I})$ Any correct form Simplification
(ii)	$\lambda(\lambda-8)(\lambda+6)=0$ Other eigenvalues are $8,\ -6$ When $\lambda=8$, $3x+5y+2z=8x$ $(\ 5x+3y-2z=8y\ )$ $2x-2y-4z=8z$ $y=x$ and $z=0$; eigenvector is $\begin{pmatrix}1\\1\\0\end{pmatrix}$ When $\lambda=-6$, $3x+5y+2z=-6x$ $5x+3y-2z=-6y$ $y=-x,\ z=-2x$; eigenvector is $\begin{pmatrix}1\\-1\\-2\end{pmatrix}$	M1 A1 M1 M1 A1 M1 M1 A1 **8**	Solving to obtain a non-zero value Two independent equations Obtaining a non-zero eigenvector *($-5x+5y+2z=8x$ etc can earn M0M1)* Two independent equations Obtaining a non-zero eigenvector
(iii)	$\mathbf{P}=\begin{pmatrix}1&1&1\\-1&1&-1\\1&0&-2\end{pmatrix}$ $\mathbf{D}=\begin{pmatrix}0&0&0\\0&8&0\\0&0&-6\end{pmatrix}^2$ $=\begin{pmatrix}0&0&0\\0&64&0\\0&0&36\end{pmatrix}$	B1 ft M1 A1 **3**	B0 if **P** is clearly singular Order must be consistent with **P** when B1 has been earned
(iv)	$\mathbf{M}^3-2\mathbf{M}^2-48\mathbf{M}=\mathbf{0}$ $\mathbf{M}^3=2\mathbf{M}^2+48\mathbf{M}$ $\mathbf{M}^4=2\mathbf{M}^3+48\mathbf{M}^2$ $=2(2\mathbf{M}^2+48\mathbf{M})+48\mathbf{M}^2$ $=52\mathbf{M}^2+96\mathbf{M}$	M1 M1 A1	

MEI FP2 Jan 2008

3 (i)	Characteristic equation is $(7-\lambda)(-1-\lambda)+12=0$ $\lambda^2-6\lambda+5=0$	M1		
	$\lambda=1,\ 5$	A1A1		
	When $\lambda=1$, $\begin{pmatrix}7 & 3\\ -4 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$	M1		or $\begin{pmatrix}6 & 3\\ -4 & -2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ *can be awarded for either eigenvalue*
	$7x+3y=x$ $-4x-y=y$	M1		Equation relating *x* and *y*
	$y=-2x$, eigenvector is $\begin{pmatrix}1\\ -2\end{pmatrix}$	A1		or any (non-zero) multiple
	When $\lambda=5$, $\begin{pmatrix}7 & 3\\ -4 & -1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=5\begin{pmatrix}x\\ y\end{pmatrix}$ $7x+3y=5x$ $-4x-y=5y$	M1		
	$y=-\frac{2}{3}x$, eigenvector is $\begin{pmatrix}3\\ -2\end{pmatrix}$	A1	8	*SR* $(\mathbf{M}-\lambda\mathbf{I})\mathbf{x}=\lambda\mathbf{x}$ can earn M1A1A1M0M1A0M1A0
(ii)	$\mathbf{P}=\begin{pmatrix}1 & 3\\ -2 & -2\end{pmatrix}$	B1 ft		B0 if **P** is singular
	$\mathbf{D}=\begin{pmatrix}1 & 0\\ 0 & 5\end{pmatrix}$	B1 ft	2	For B2, the order must be consistent
(iii)	$\mathbf{M}=\mathbf{P}\mathbf{D}\mathbf{P}^{-1}$	M1		*May be implied*
	$\mathbf{M}^n=\mathbf{P}\mathbf{D}^n\mathbf{P}^{-1}$	M1		
	$=\mathbf{P}\begin{pmatrix}1 & 0\\ 0 & 5^n\end{pmatrix}\mathbf{P}^{-1}$	A1 ft		*Dependent on M1M1*
	$=\begin{pmatrix}1 & 3\\ -2 & -2\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 5^n\end{pmatrix}\frac{1}{4}\begin{pmatrix}-2 & -3\\ 2 & 1\end{pmatrix}$	B1 ft		For $\mathbf{P}^{-1}$
	$=\begin{pmatrix}1 & 3\times5^n\\ -2 & -2\times5^n\end{pmatrix}\frac{1}{4}\begin{pmatrix}-2 & -3\\ 2 & 1\end{pmatrix}$			or $\begin{pmatrix}1 & 3\\ -2 & -2\end{pmatrix}\frac{1}{4}\begin{pmatrix}-2 & -3\\ 2\times5^n & 5^n\end{pmatrix}$
	$=\frac{1}{4}\begin{pmatrix}-2+6\times5^n & -3+3\times5^n\\ 4-4\times5^n & 6-2\times5^n\end{pmatrix}$	M1		Obtaining at least one elemen in a product of three matrices
	$a=-\frac{1}{2}+\frac{3}{2}\times5^n$	A1 ag		
	$b=-\frac{3}{4}+\frac{3}{4}\times5^n$ $c=1-5^n$ $\quad d=\frac{3}{2}-\frac{1}{2}\times5^n$	A2	8	Give A1 for one of *b, c, d* correct *SR* If $\mathbf{M}^n=\mathbf{P}^{-1}\mathbf{D}^n\mathbf{P}$ is used, max marks are M0M1A0B1M1A0A1 (*d* should be correct) *SR* If their **P** is singular, max marks are M1M1A1B0M0

MEI FP2 June 2008

3 (i)	$\mathbf{Q}^{-1} = \dfrac{1}{k-3}\begin{pmatrix} -1 & k+2 & -1 \\ 1 & 4-3k & k-2 \\ 1 & -5 & 1 \end{pmatrix}$ When $k=4$, $\mathbf{Q}^{-1} = \begin{pmatrix} -1 & 6 & -1 \\ 1 & -8 & 2 \\ 1 & -5 & 1 \end{pmatrix}$	M1 A1 M1 A1 M1 A1 **6**	Evaluation of determinant *(must involve k)* For $(k-3)$ Finding at least four cofactors *(including one involving k)* Six signed cofactors correct *(including one involving k)* Transposing and dividing by det *Dependent on previous M1M1* $\mathbf{Q}^{-1}$ correct (in terms of *k*) and result for $k=4$ stated After 0, SC1 for $\mathbf{Q}^{-1}$ when $k=4$ obtained correctly with some working
(ii)	$\mathbf{P} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}$, $\mathbf{D} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$ $\mathbf{M} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$ $= \begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 6 & -1 \\ 1 & -8 & 2 \\ 1 & -5 & 1 \end{pmatrix}$ $= \begin{pmatrix} 2 & 1 & 12 \\ 1 & 0 & 3 \\ 3 & -1 & 6 \end{pmatrix}\begin{pmatrix} -1 & 6 & -1 \\ 1 & -8 & 2 \\ 1 & -5 & 1 \end{pmatrix}$ $= \begin{pmatrix} 11 & -56 & 12 \\ 2 & -9 & 2 \\ 2 & -4 & 1 \end{pmatrix}$	B1B1 B2 M1 A2 **7**	For B2, order must be consistent Give B1 for $\mathbf{M} = \mathbf{P}^{-1}\mathbf{D}\mathbf{P}$ or $\begin{pmatrix} 2 & -1 & 4 \\ 1 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}\begin{pmatrix} -1 & 6 & -1 \\ -1 & 8 & -2 \\ 3 & -15 & 3 \end{pmatrix}$ Good attempt at multiplying two matrices (no more than 3 errors), leaving third matrix in correct position Give A1 for five elements correct Correct **M** implies B2M1A2 5-8 elements correct implies B2M1A1
(iii)	Characteristic equation is $(\lambda-1)(\lambda+1)(\lambda-3)=0$ $\lambda^3 - 3\lambda^2 - \lambda + 3 = 0$ $\mathbf{M}^3 = 3\mathbf{M}^2 + \mathbf{M} - 3\mathbf{I}$ $\mathbf{M}^4 = 3\mathbf{M}^3 + \mathbf{M}^2 - 3\mathbf{M}$ $= 3(3\mathbf{M}^2 + \mathbf{M} - 3\mathbf{I}) + \mathbf{M}^2 - 3\mathbf{M}$ $= 10\mathbf{M}^2 - 9\mathbf{I}$ $a=10,\ b=0,\ c=-9$	B1 M1 A1 M1 A1 **5**	In any correct form *(Condone omission of =0)* **M** satisfies the characteristic equation Correct expanded form *(Condone omission of* **I** *)*

MEI FP2 Jan 09

)(i)	Characteristic equation is $(0.2-\lambda)(0.7-\lambda)-0.24=0$	M1	
	$\Rightarrow \lambda^2-0.9\lambda-0.1=0$		
	$\Rightarrow \lambda=1, -0.1$	A1	
	When $\lambda=1$, $\begin{pmatrix}-0.8 & 0.8\\ 0.3 & -0.3\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$		$(\mathbf{M}-\lambda\mathbf{I})\boldsymbol{x}=\boldsymbol{x}$ M0 below
	$\Rightarrow -0.8x+0.8y=0,\ 0.3x-0.3y=0$	M1	At least one equation relating x and y
	$\Rightarrow x-y=0$, eigenvector is $\begin{pmatrix}1\\ 1\end{pmatrix}$ o.e.	A1	
	When $\lambda=-0.1$, $\begin{pmatrix}0.3 & 0.8\\ 0.3 & 0.8\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$		
	$\Rightarrow 0.3x+0.8y=0$	M1	At least one equation relating x and y
	$\Rightarrow$ eigenvector is $\begin{pmatrix}8\\ -3\end{pmatrix}$ o.e.	A1 **6**	
(ii)	$\mathbf{Q}=\begin{pmatrix}1 & 8\\ 1 & -3\end{pmatrix}$	B1ft	B0 if **Q** is singular. Must label correctly
	$\mathbf{D}=\begin{pmatrix}1 & 0\\ 0 & -0.1\end{pmatrix}$	B1ft B1	If order consistent. Dep on B1B1 earned
		3	**1**:

MEI FP2 Jan 11

3 (i)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (1-\lambda)[(3-\lambda)(1-\lambda)+8]$ $+4[2(1-\lambda)-2]+5[8+(3-\lambda)]$ $=(1-\lambda)(\lambda^2-4\lambda+11)+4(-2\lambda)+5(11-\lambda)$ $=-\lambda^3+5\lambda^2-15\lambda+11-8\lambda+55-5\lambda=0$ $\Rightarrow \lambda^3-5\lambda^2+28\lambda-66=0$	M1 A1 M1 A1 (ag) **4**	Obtaining $\det(\mathbf{M}-\lambda\mathbf{I})$ Any correct form Simplification www, but condone omission of = 0
(ii)	$\lambda^3-5\lambda^2+28\lambda-66=0$ $\Rightarrow(\lambda-3)(\lambda^2-2\lambda+22)=0$ $\lambda^2-2\lambda+22=0 \Rightarrow b^2-4ac=-84$ so no other real eigenvalues	M1 A1 M1 A1 **4**	Factorising and obtaining a quadratic. If M0, give B1 for substituting $\lambda = 3$ Correct quadratic Considering discriminant o.e. Conclusion from correct evidence www
(iii)	$\lambda=3 \Rightarrow \begin{pmatrix}-2 & -4 & 5\\ 2 & 0 & -2\\ -1 & 4 & -2\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$ $\Rightarrow -2x-4y+5z=0$ $2x-2z=0$ $-x+4y-2z=0$ $\Rightarrow x=z=k,\ y=\frac{3}{4}k$ $\Rightarrow$ eigenvector is $\begin{pmatrix}4\\3\\4\end{pmatrix}$ $\Rightarrow$ eigenvector with unit length is $\mathbf{v}=\frac{1}{\sqrt{41}}\begin{pmatrix}4\\3\\4\end{pmatrix}$ Magnitude of $\mathbf{M}^n\mathbf{v}$ is 3^n	 M1 M1 A1 B1 B1 **5**	 Two independent equations Obtaining a non-zero eigenvector Must be a magnitude
(iv)	$\lambda^3-5\lambda^2+28\lambda-66=0$ $\Rightarrow \mathbf{M}^3-5\mathbf{M}^2+28\mathbf{M}-66\mathbf{I}=\mathbf{0}$ $\Rightarrow \mathbf{M}^2-5\mathbf{M}+28\mathbf{I}-66\mathbf{M}^{-1}=\mathbf{0}$ $\Rightarrow \mathbf{M}^{-1}=\frac{1}{66}(\mathbf{M}^2-5\mathbf{M}+28\mathbf{I})$	 M1 M1 A1 **3**	 Use of Cayley-Hamilton Theorem Multiplying by $\mathbf{M}^{-1}$ and rearranging Must contain **I** **16**

MEI FP2 June 2011

3 (i)	$\det(\mathbf{M}) = 1(16-12) + 1(20-18) + k(10-12)$ $= 6 - 2k$ $\Rightarrow$ no inverse if $k = 3$ $\mathbf{M}^{-1} = \frac{1}{6-2k}\begin{pmatrix} 4 & 4+2k & -6-4k \\ -2 & 4-3k & 5k-6 \\ -2 & -5 & 9 \end{pmatrix}$	M1 A1 A1 M1 A1 M1 A1	7	Obtaining det(**M**) in terms of k Accept $k \neq 3$ after correct determinant Evaluating at least four cofactors (including one involving k) Six signed cofactors correct (including one involving k) Transposing and dividing by det(**M**). Dependent on previous M1M1
(ii)	$\begin{pmatrix} 1 & -1 & 3 \\ 5 & 4 & 6 \\ 3 & 2 & 4 \end{pmatrix}\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$	M1 A1	2	Setting $k = 3$ and multiplying
(iii)	$\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}$ is an eigenvector corresponding to an eigenvalue of 1	B1 B1	2	For credit here, 2/2 scored in (ii) Accept "invariant point"

MEI FP2 Jan 2012

(i)		$\mathbf{M}-\lambda\mathbf{I}=\begin{pmatrix}3-\lambda & -1 & 2\\ -4 & 3-\lambda & 2\\ 2 & 1 & -1-\lambda\end{pmatrix}$			
		$\det(\mathbf{M}-\lambda\mathbf{I})=(3-\lambda)[(3-\lambda)(-1-\lambda)-2]$	M1	Obtaining $\det(\mathbf{M}-\lambda\mathbf{I})$	
		$+1[-4(-1-\lambda)-4]+2[-4-2(3-\lambda)]$	A1	Any correct form	
		$=(3-\lambda)(\lambda^2-2\lambda-5)+4\lambda+2(2\lambda-10)$	M1	Multiplying out. Dep. on first M1	
		$=-\lambda^3+5\lambda^2-\lambda-15+4\lambda+4\lambda-20$			
		$\Rightarrow \lambda^3-5\lambda^2-7\lambda+35=0$	E1		Answer given
			[4]		
(ii)		$\lambda^3-5\lambda^2-7\lambda+35=0$	M1	Factorising, obtaining a quadratic	If M0, give B1 for substituting $\lambda=5$
		$\Rightarrow(\lambda-5)(\lambda^2-7)=0$	A1	Correct quadratic	
			M1	Solving quadratic	
		$\lambda=\pm\sqrt{7}$	A1		Allow 2.65 or better
			[4]		
(iii)		$\lambda=5\Rightarrow\begin{pmatrix}-2 & -1 & 2\\ -4 & -2 & 2\\ 2 & 1 & -6\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$			
		$\Rightarrow -2x-y+2z=0$			
		$-4x-2y+2z=0$	M1	Two independent equations	Need to multiply out, unless implied by later work
		$2x+y-6z=0$			
		$\Rightarrow z=0,\ y=-2x$	M1	Obtaining a non-zero eigenvector	
		$\Rightarrow$ eigenvector is $\begin{pmatrix}1\\ -2\\ 0\end{pmatrix}$	A1		
		$\Rightarrow$ eigenvector of unit length is $\frac{1}{\sqrt{5}}\begin{pmatrix}1\\ -2\\ 0\end{pmatrix}$	A1ft	$\frac{1}{\sqrt{5}}$ f.t. their eigenvector	
		$\mathbf{M}^2\mathbf{v}$ has magnitude 25 in direction of $\mathbf{v}$			
		$\mathbf{M}^{-1}\mathbf{v}$ has magnitude $\frac{1}{5}$ in direction of $\mathbf{v}$	B1	Both magnitudes c.a.o.	
			B1	Directions c.a.o.	May be given as column vectors
			[6]		
(iv)		$\lambda^3-5\lambda^2-7\lambda+35=0$			
		$\Rightarrow \mathbf{M}^3-5\mathbf{M}^2-7\mathbf{M}+35\mathbf{I}=\mathbf{0}$	M1	Using Cayley-Hamilton Theorem	Condone omitted $\mathbf{I}$
		$\Rightarrow \mathbf{M}^4=5\mathbf{M}^3+7\mathbf{M}^2-35\mathbf{M}$	A1	Correct expression involving $\mathbf{M}^4$ and non-negative powers of $\mathbf{M}$	
		$=5(5\mathbf{M}^2+7\mathbf{M}-35\mathbf{I})+7\mathbf{M}^2-35\mathbf{M}$	M1	Substituting for $\mathbf{M}^3$ and obtaining expression in required form	
		$=32\mathbf{M}^2-175\mathbf{I}$	A1	$a=32,\ b=0,\ c=-175$	
			[4]		

MEI FP2 Jan 2013

(i)		$\mathbf{M}-\lambda\mathbf{I}=\begin{pmatrix}1-\lambda & 3 & 0\\ 3 & -2-\lambda & -1\\ 0 & -1 & 1-\lambda\end{pmatrix}$			
		$\det(\mathbf{M}-\lambda\mathbf{I})$	M1	Forming $\det(\mathbf{M}-\lambda\mathbf{I})$	
		$=(1-\lambda)[(-2-\lambda)(1-\lambda)-1]-3[3(1-\lambda)]$	A1	Any correct form	Sarrus: $(1-\lambda)^2(-2-\lambda)-10(1-\lambda)$
		$=(1-\lambda)(\lambda^2+\lambda-3)-9(1-\lambda)$			or e.g. $\lambda-1+(1-\lambda)(\lambda^2+\lambda-11)$
		$\Rightarrow\lambda^3-13\lambda+12=0$	A1(ag) **[3]**	Condone omission of 0	
(ii)		$(\lambda-1)(\lambda^2+\lambda-12)=0$	M1	Factorising as far as quadratic	Allow one error
		$\Rightarrow(\lambda-1)(\lambda-3)(\lambda+4)=0$	A1		
		$\Rightarrow$ eigenvalues are 1, 3, −4	A1		
		$\lambda=1:\ \begin{pmatrix}0 & 3 & 0\\ 3 & -3 & -1\\ 0 & -1 & 0\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$	M2	For any one of $\lambda=1,3,-4$ Obtaining two independent equations	From which an eigenvector could be found
			M1	Obtaining a non-zero eigenvector	
		$\Rightarrow y=0,\ 3x-z=0$	A1	o.e.	Allow e.g. $3y=0,\ 3x-3y-z=0$
		$\Rightarrow$ eigenvector is $\begin{pmatrix}1\\ 0\\ 3\end{pmatrix}$	A1		
		$\lambda=3:\ \begin{pmatrix}-2 & 3 & 0\\ 3 & -5 & -1\\ 0 & -1 & -2\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$			
		$\Rightarrow-2x+3y=0,\ -y-2z=0$	A1	o.e.	
		$\Rightarrow y=-2z,\ x=-3z$			
		$\Rightarrow$ eigenvector is $\begin{pmatrix}-3\\ -2\\ 1\end{pmatrix}$	A1		
		$\lambda=-4:\ \begin{pmatrix}5 & 3 & 0\\ 3 & 2 & -1\\ 0 & -1 & 5\end{pmatrix}\begin{pmatrix}x\\ y\\ z\end{pmatrix}=\begin{pmatrix}0\\ 0\\ 0\end{pmatrix}$			
		$\Rightarrow5x+3y=0,\ -y+5z=0$	A1	o.e.	
		$\Rightarrow y=5z,\ x=-3z$			
		$\Rightarrow$ eigenvector is $\begin{pmatrix}-3\\ 5\\ 1\end{pmatrix}$	A1 **[12]**		
(iii)		E.g. $\mathbf{P}=\begin{pmatrix}1 & -3 & -3\\ 0 & -2 & 5\\ 3 & 1 & 1\end{pmatrix}$	B1	Use of eigenvectors (ft) as columns	
		$\mathbf{D}=\begin{pmatrix}1 & 0 & 0\\ 0 & 3^n & 0\\ 0 & 0 & (-4)^n\end{pmatrix}$	M1	Use of 1, 3, −4 (ft) in correct order	n not required for M1
			A1 **[3]**	Power n	-4^n A0

MEI FP2 June 2015

(i)	$\det(\mathbf{M}-\lambda\mathbf{I}) = (5-\lambda)((-3-\lambda)(4-\lambda)+2)$ $+(4(4-\lambda)+4)$ $+3(4-2(-3-\lambda))$ Simplify to $\lambda^3-6\lambda^2-7\lambda=0$ Solve to $\lambda=-1, 0, 7$	M1A1 A1 A1 A1 M1A1 **[7]**	M1 for attempt at $\det(\mathbf{M}-\lambda\mathbf{I})$ A1 each term correct A0 if ' = 0 ' never appears M1 for eigenvalues are roots of char eqn	
(ii)	 Show that $\mathbf{M}(1\ \ 3\ \ -1)^T = (-1\ \ -3\ \ 1)^T$ Show that $\mathbf{M}(1\ \ 2\ \ -1)^T = (0\ \ 0\ \ 0)^T$ Obtain equations $-2a+3c=1, 2a-c=5$ or equivalent Solve to obtain $(4\ \ 1\ \ 3)^T$	M1 A1 A1 B1 B1 **[5]**	For clear evidence of understanding A1 each calculation FT Two correct equations CAO *Accept a = 4, c = 3*	e.g. Just finding eigenvector for $\lambda=7$ would earn M1A0A0B1B1
(iii)	$P=\begin{pmatrix}1&1&4\\3&2&1\\-1&-1&3\end{pmatrix}\quad D=\begin{pmatrix}-1&0&0\\0&0&0\\0&0&7\end{pmatrix}$ $\mathbf{M}^4=\mathbf{PD}^4\mathbf{P}^{-1}$ where $\mathbf{D}^4=\text{diag}(1\ \ 0\ \ 2401)$	B1B1 B1 **[3]**	FT CAO	*For B2, order must be consistent*
(iv)	C-H: $\mathbf{M}^3=6\mathbf{M}^2+7\mathbf{M}$ $\mathbf{M}^4=6\mathbf{M}^3+7\mathbf{M}^2$ $=6(6\mathbf{M}^2+7\mathbf{M})+7\mathbf{M}^2$ $=43\mathbf{M}^2+42\mathbf{M}$	M1 A1 A1 **[3]** **[18]**	 CAO CAO	