## Recurrence Relations Past Paper Pack – Mark Schemes

Edexcel Further Pure 2 June 2019

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|---|---|---|---|
| **3(a)** | $V_{n+2} = V_{n+1} + kV_n$ | B1 | 3.3 |
| | | **(1)** | |
| **(b)** | $\lambda^2 - \lambda - 0.24 = 0 \Rightarrow \lambda = ...(1.2, -0.2)$ | M1 | 1.1b |
| | $V_n = a(1.2)^n + b(-0.2)^n$ | A1 | 2.2a |
| | $65 = a(1.2)^1 + b(-0.2)^1$ **and** $71 = a(1.2)^2 + b(-0.2)^2$ | B1ft | 3.4 |
| | E.g. $\left.\begin{array}{l} 78 = 1.44a - 0.24b \\ 71 = 1.44a + 0.04b \end{array}\right\} \Rightarrow 7 = -0.28b \Rightarrow b = ...$ | M1 | 2.1 |
| | $a = 50, b = -25 \Rightarrow V_n = 50(1.2)^n - 25(-0.2)^n$ * | A1* | 1.1b |
| | | **(5)** | |
| **(c)** | $50(1.2)^N > 10^6 \Rightarrow N = ...$ | M1 | 3.1b |
| | $\Rightarrow N = 55$ i.e. month 55 | A1 | 3.2a |
| | | **(2)** | |

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| Auxiliary equation is $9r^2-12r+4=0$, so $r=...$ | **M1** | 1.1b |
| $(3r-2)^2=0\Rightarrow r=\frac{2}{3}$ is repeated root. | **A1** | 1.1b |
| Complementary function is $x_n=(A+Bn)\left(\frac{2}{3}\right)^n$ or $A\left(\frac{2}{3}\right)^n+Bn\left(\frac{2}{3}\right)^n$ | **M1** | 2.2a |
| Try particular solution<br>$y_n=an+b\Rightarrow 9(a(n+2)+b)-12(a(n+1)+b)+4(an+b)=3n$ | **M1** | 2.1 |
| $\Rightarrow an+6a+b=3n\Rightarrow a=...,b=...$ | **dM1** | 1.1b |
| $a=3,b=-18$ | **A1** | 1.1b |
| General solution is $u_n=x_n+y_n=(A+Bn)\left(\frac{2}{3}\right)^n+3n-18$ | **B1ft** | 2.2a |
| $\left.\begin{array}{l}u_1=1\Rightarrow 1=\left(\frac{2}{3}\right)(A+B)-15\\ u_2=4\Rightarrow 4=\left(\frac{4}{9}\right)(A+2B)-12\end{array}\right\\}A=...,B=...$ | **M1** | 2.1 |
| $u_n=12(n+1)\left(\frac{2}{3}\right)^n+3n-18$ oe | **A1** | 1.1b |
| | **(9)** | |

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| **8(a)** | $u_1 = 1$ as there is only one way to go up one step | B1 | 2.4 |
| | $u_2 = 2$ as there are two ways: one step then one step or two steps | B1 | 2.4 |
| | If first move is one step then can climb the other $(n-1)$ steps in $u_{n-1}$ ways. If first move is two steps can climb the other $(n-2)$ steps in $u_{n-2}$ ways so $u_n = u_{n-1} + u_{n-2}$ | B1 | 2.4 |
| | | **(3)** | |
| **(b)** | Sequence begins 1, 2, 3, 5, 8, 13, 21, 34,… so 34 ways of climbing 8 steps | B1 | 1.1b |
| | | **(1)** | |
| **(c)** | To find general term use $u_n = u_{n-1} + u_{n-2}$ gives $\lambda^2 = \lambda + 1$ | M1 | 2.1 |
| | This has roots $\frac{1 \pm \sqrt{5}}{2}$ | A1 | 1.1b |
| | So general form is $A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n$ | M1 | 2.2a |
| | Uses initial conditions to find $A$ and $B$ reaching two equations in $A$ and $B$ | M1 | 1.1b |
| | Obtains $A = \left(\frac{1+\sqrt{5}}{2\sqrt{5}}\right)$ and $B = -\left(\frac{1-\sqrt{5}}{2\sqrt{5}}\right)$ and so when $n = 400$ obtains $\frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right)^{401} - \left(\frac{1-\sqrt{5}}{2}\right)^{401}\right]$ * | A1* | 1.1b |
| | | **(5)** | |

**(9 marks)**