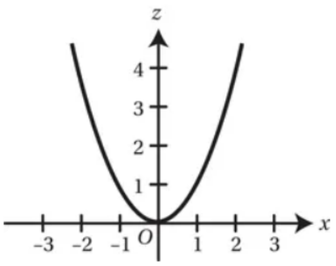
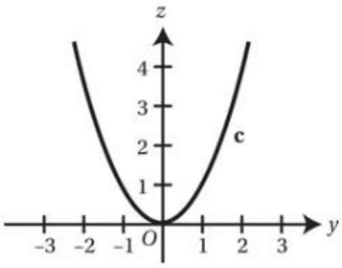
ANSWERS

Exercise 5A

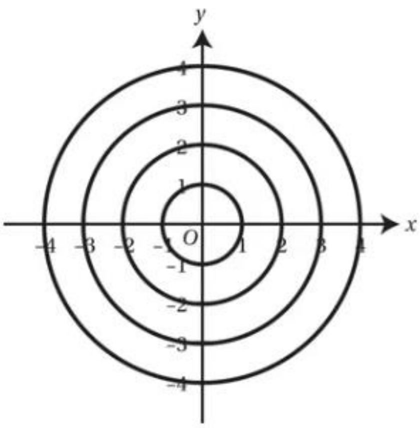
1 a i $z = x^2$



ii $z = y^2$

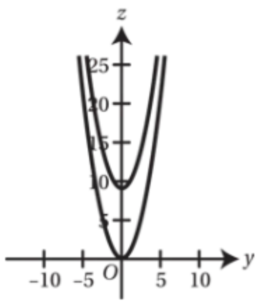


b $x^2 + y^2 = c$



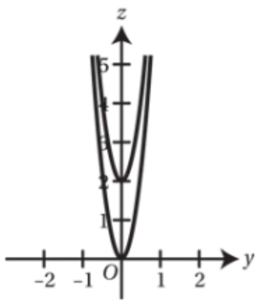
2 **a** **i** $z = x^2$

$z = x^2 + 9$

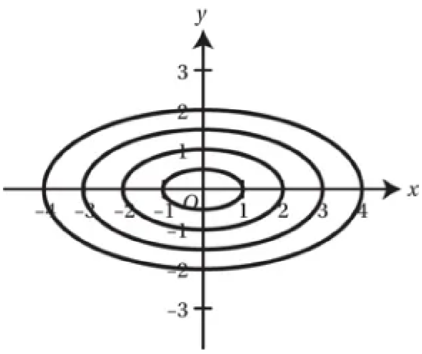


ii $z = 9y^2$

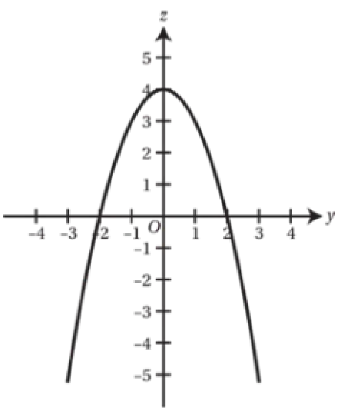
$z = 9y^2 + 4$



b $x^2 + 9y^2 = c$

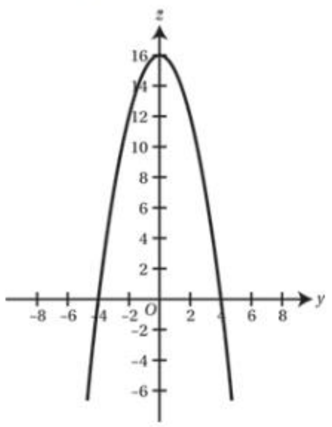


3 **a** $z = x^2 - 4$



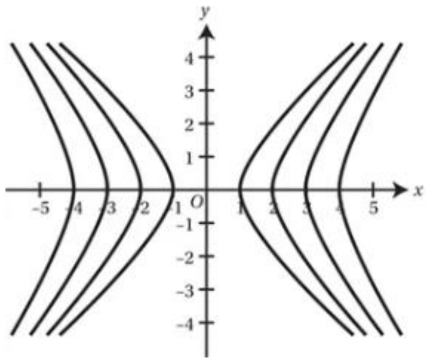
$(\pm 2,\ 2,\ 0)$

b $z = 16 - y^2$

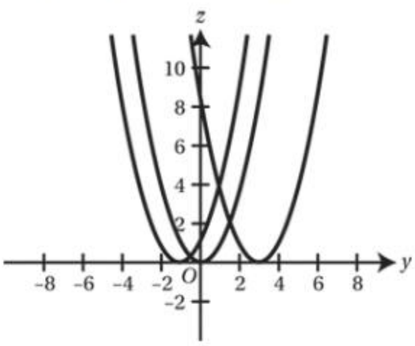


$(4, \pm 3,\ 7)$

c $x^2 - y^2 = c$

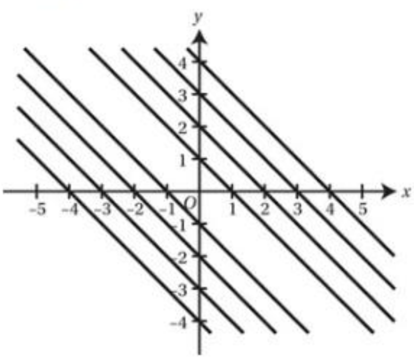


4 a $z = x^2, z = (x+1)^2, z = (x-3)^2$

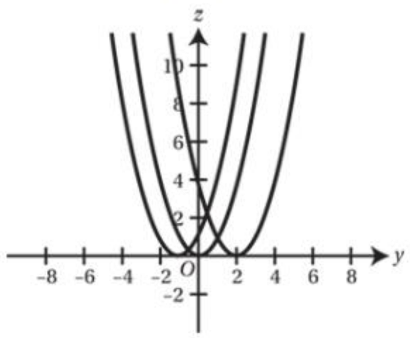


$(0,0,0), (-1,1,0),\ (3,-3,0)$

b $(x+y)^2 = c$

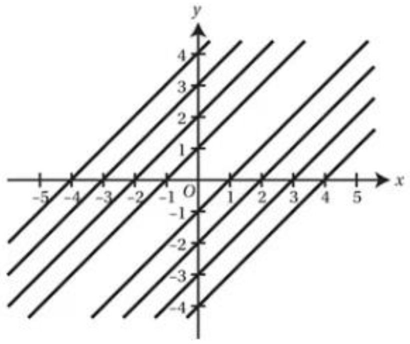


5 a $z = x^2, z = (x-2)^2, z = (x+1)^2$



$(0, \pm 2, 4), (2, 0, 4)$ or $(2, 4, 4)$, $(-1, 1, 4)$ or $(-1, -3, 4)$

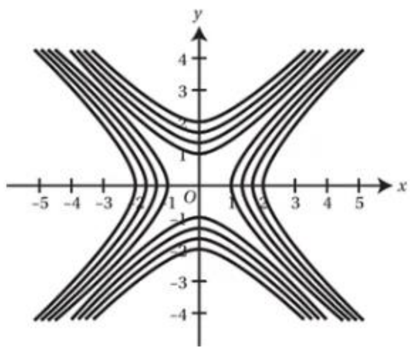
b $(x-y)^2 = c$



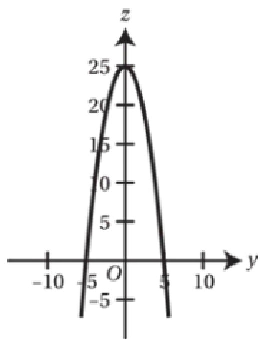
6 a Yes

b $\left(5^2 - 4^2\right)^2 = 81 > 0$ so above the surface.

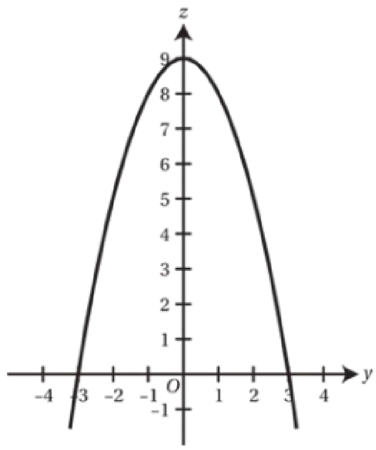
c $\left(x^2 - y^2\right)^2 = c$



7 a i $z = 25 - x^2$

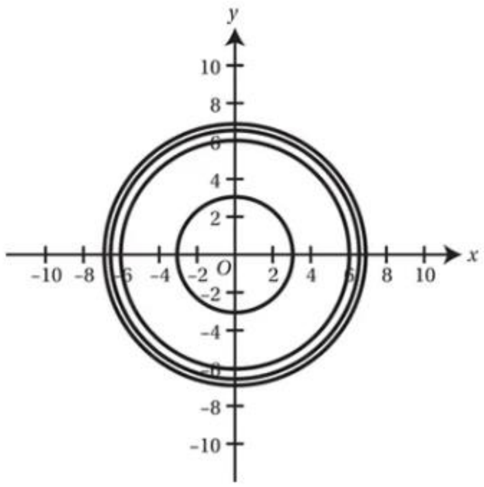


ii $z = 9 - y^2$

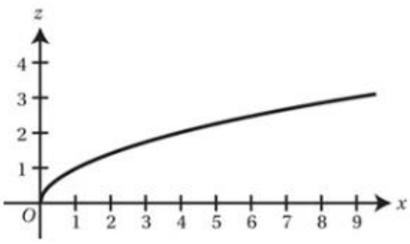


b e.g. $(\pm 3, \pm 4, 0)$

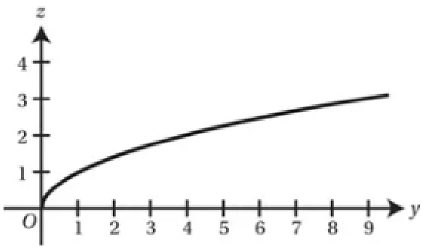
c $25 - (x^2 + y^2) = c$



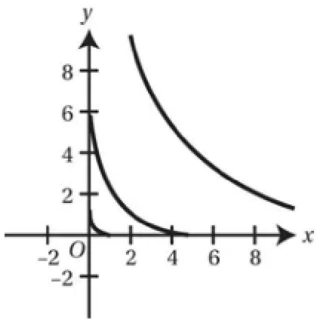
8 a i $z = \sqrt{x}$



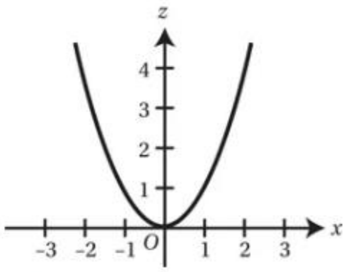
ii $z = \sqrt{y}$



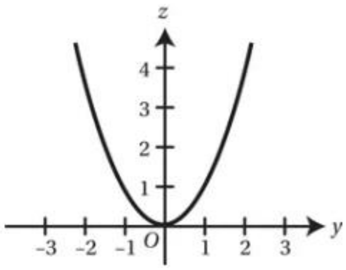
b $\sqrt{x} + \sqrt{y} = c$



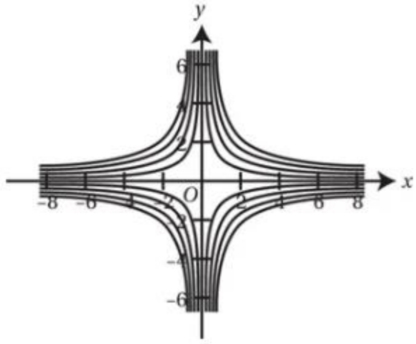
9 **a** **i** $z = x^2$



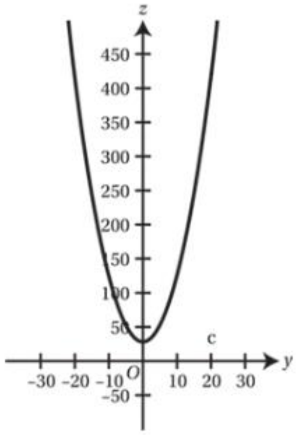
ii $z = y^2$



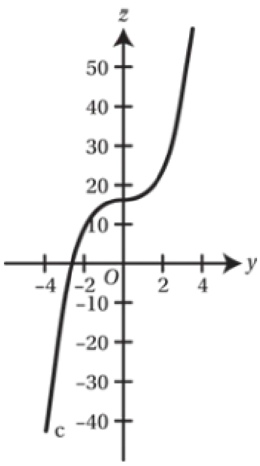
b $x^2y^2 = c$



10 a i $z = x^2 + 27$



ii $z = 16 + y^3$

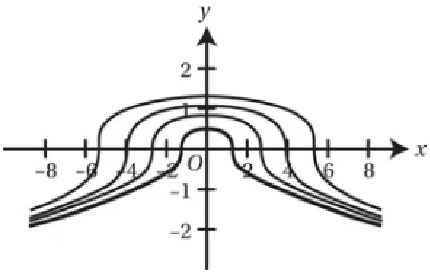


b $x = -4 \Rightarrow z = 43$

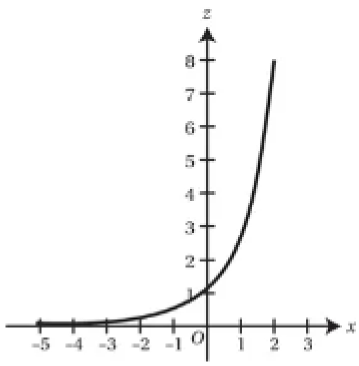
$y = 3 \Rightarrow z = 43$

i.e. intersect at $(-4, 3, 43)$

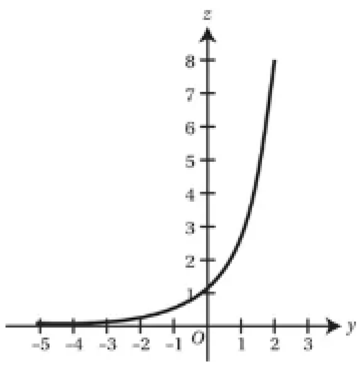
c $x^2 + y^3 = c$



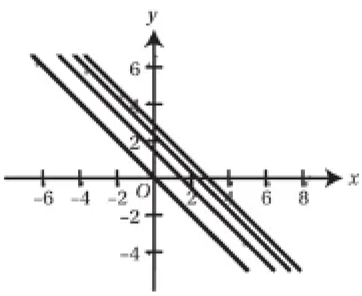
11 a i $z = e^x$



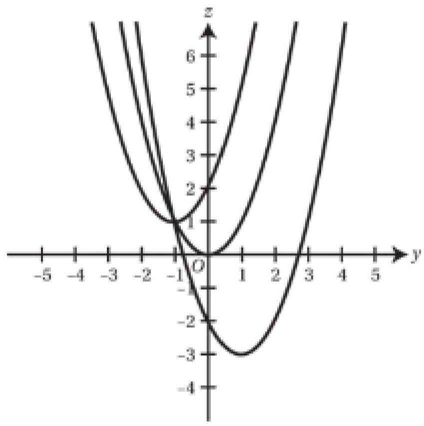
ii $z = e^y$



b $e^{x+y} = c$

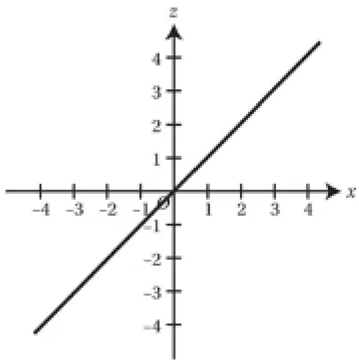


12 a $z = x^2, z = x^2 - 2x - 2, z = x^2 + 2x + 2$

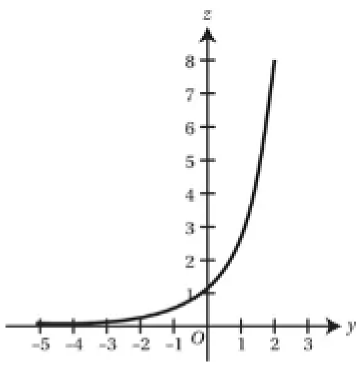


b $z = x^2, z = x^2 - 2x - 2$

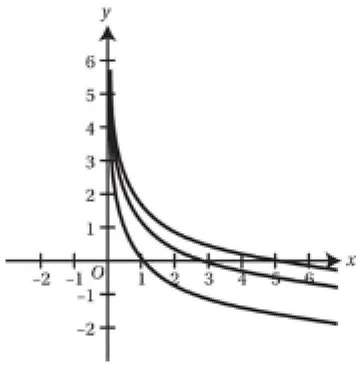
13 a i $z = x$



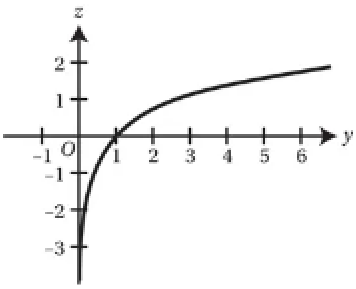
ii $z = e^y$



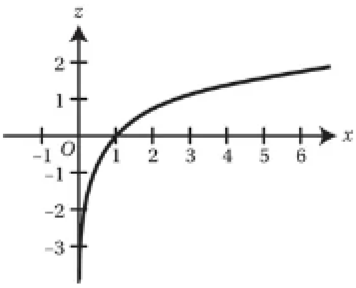
b $xe^y = c$



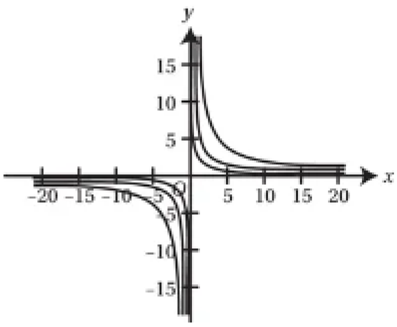
14 a i $z = \ln(x)$



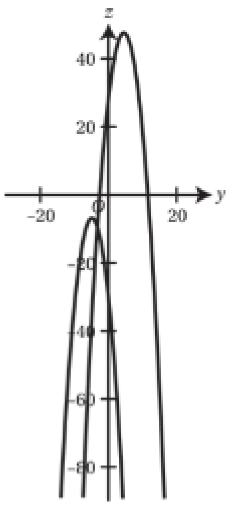
ii $z = \ln(y)$



b $\ln(xy) = c$

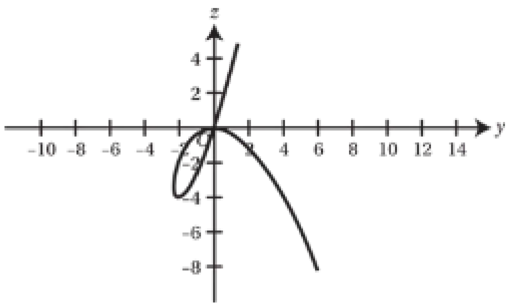


15 a $z = 27 + 9y - y^2, z = -27 - 9y - y^2$

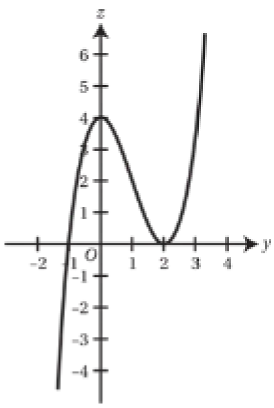


b $z = 27 + 9y - y^2$

c



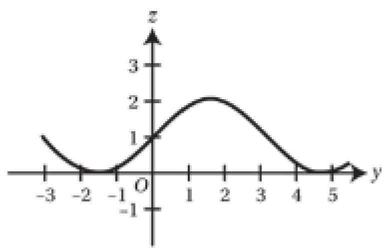
16 a $z = x^3 - 3x^2 + 4$



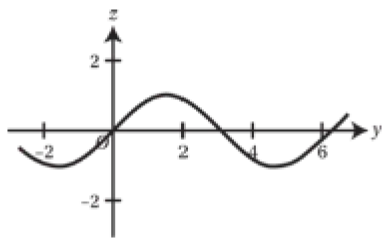
b $(-1, -1, 0), (2, -1, 0)$

c $(0, -1, 4), (2, -1, 0)$

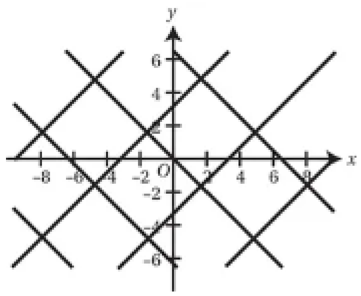
17 a **i** $z = \sin(x) + 1$



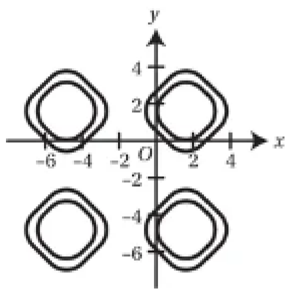
ii $z = \sin(y)$



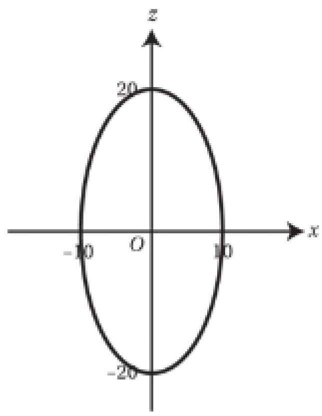
b $\sin(x) + \sin(y) = 0$



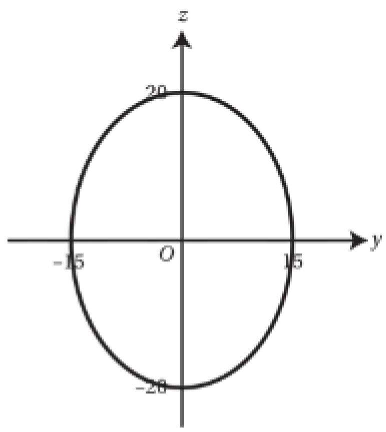
c $\sin(x) + \sin(y) = c, c = 0.5, 1$



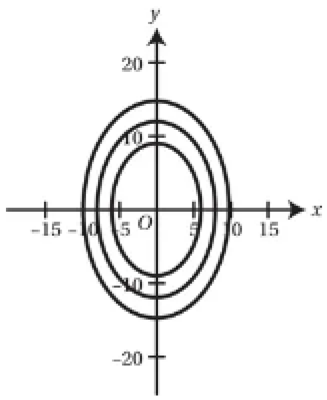
18 a i $\frac{x^2}{4} + \frac{z^2}{16} = 25$



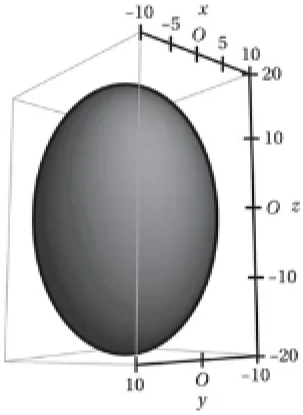
ii $\frac{y^2}{9} + \frac{z^2}{16} = 25$



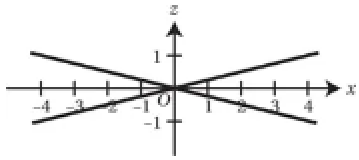
b $\frac{x^2}{4} + \frac{y^2}{9} + \frac{c^2}{16} = 25$



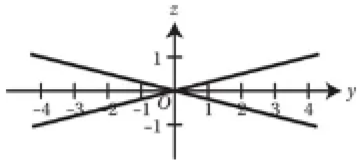
This is an ellipsoid:



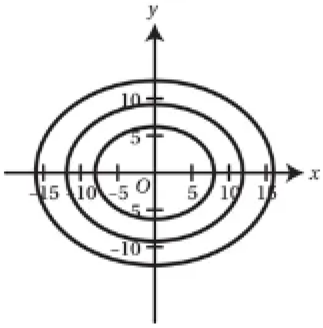
9 a i $z^2 = \frac{x^2}{16}$



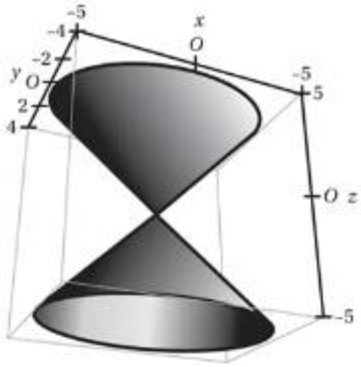
ii $z^2 = \frac{y^2}{9}$



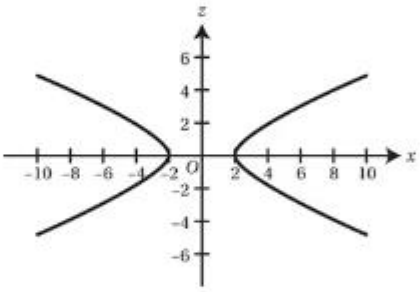
b $c^2 = \frac{x^2}{16} + \frac{y^2}{9}$



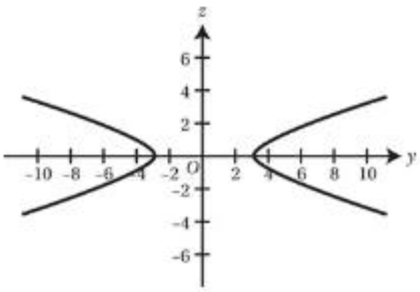
This is an elliptic cone:



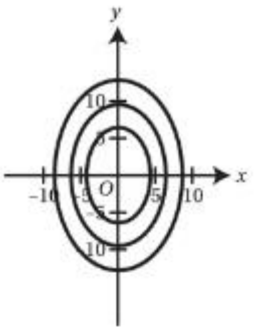
20 a i $z^2 = \frac{x^2}{4} - 1$

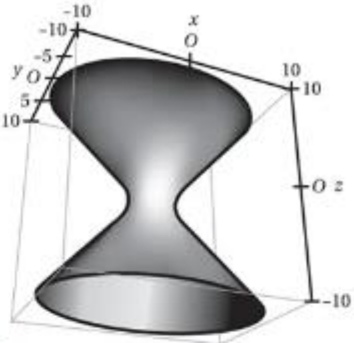


ii $z^2 = \frac{y^2}{9} - 1$



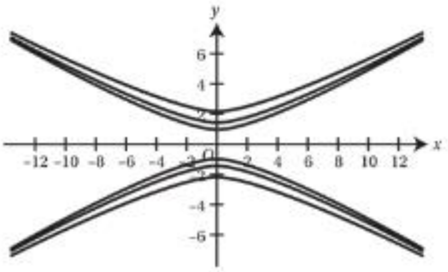
b $x^2 + y^2 = c$



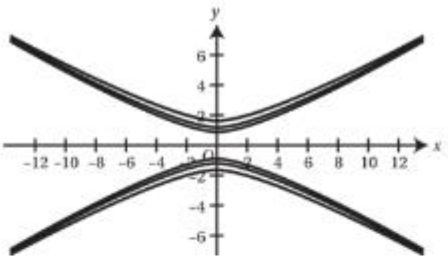


hyperboloid of one sheet

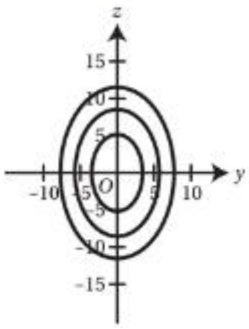
21 a i

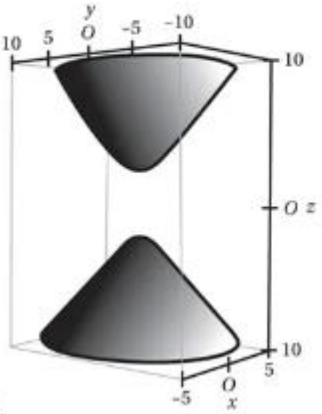


ii



b $z^2 = \frac{x^2}{4} + \frac{y^2}{9} + 1$





hyperboloid of two sheets

Exercise 5B

1 **a** $f_x = 2x$

b $f_y = 2y$

c $f_{xx} = 2$

d $f_{yy} = 2$

Proof

2 **a** $f_x = 2xy^2$

b $f_y = 2x^2y$

c $f_{xx} = 2y^2$

d $f_{yy} = 2x^2$

Proof

3 **a** $f_x = 3x^2 + 6xy^2$

b $f_y = 3y^2 + 6x^2y$

c $f_{xx} = 6x + 6y^2$

d $f_{yy} = 6y + 6x^2$

Proof

4 **a** $f_x = 2xy^2 - 4x^3$

b $f_y = 2x^2y + 4y^3$

c $f_{xx} = 2y^2 - 12x^2$

d $f_{yy} = 2x^2 + 12y^2$

$f_{xy} = f_{yx} = 4xy$, proof

5 **a** $f_x = 4x(x^2 + y^2)$

b $f_y = 4y(x^2 + y^2)$

c $f_{xx} = 12x^2 + 4y^2$

d $f_{yy} = 12y^2 + 4x^2$

Proof

6 **a** $f_x = x(x^2 - y^2)^{-\frac{1}{2}}$

b $f_y = -y(x^2 - y^2)^{-\frac{1}{2}}$

c $f_{xx} = (x^2 - y^2)^{-\frac{1}{2}} - x^2(x^2 - y^2)^{-\frac{3}{2}}$

d $f_{yy} = -(x^2 - y^2)^{-\frac{1}{2}} + y^2(x^2 - y^2)^{-\frac{3}{2}}$

Proof

7 **a** $f_x = (1 - x)(2x + 2y - x^2 - y^2)^{-\frac{1}{2}}$

b $f_y = (1 - y)(2x + 2y - x^2 - y^2)^{-\frac{1}{2}}$

c $f_{xx} = -(2x + 2y - x^2 - y^2)^{-\frac{1}{2}} - (1 - x)(2x + 2y - x^2 - y^2)^{-\frac{3}{2}}$

d $f_{yy} = -(2x + 2y - x^2 - y^2)^{-\frac{1}{2}} - (1 - y)(2x + 2y - x^2 - y^2)^{-\frac{3}{2}}$

Proof

8 **a** $f_x = (2x + 1)^{-\frac{1}{2}}(2y + 2)^{\frac{1}{3}}$

b $f_y = \frac{2}{3}(2x + 1)^{\frac{1}{2}}(2y + 2)^{-\frac{2}{3}}$

c $f_{xx} = -(2x + 1)^{-\frac{3}{2}}(2y + 3)^{\frac{1}{3}}$

d $f_{yy} = -\frac{8}{3}(2x + 1)^{\frac{1}{2}}(2y + 2)^{-\frac{5}{3}}$

Proof

9 **a** $f_x = \frac{2x}{y^2+2}$

b $f_y = -\frac{2y(x^2+2)}{(y^2+2)^2}$

c $f_{xx} = \frac{2}{y^2+2}$

d $f_{yy} = -\frac{2(x^2+2)}{(y^2+2)^2} + \frac{(x^2+2)(6y^2-4)}{(y^2+2)^3}$

Proof

10 **a** $f_x = e^{x+y}$

b $f_y = e^{x+y}$

c $f_{xx} = e^{x+y}$

d $f_{yy} = e^{x+y}$

Proof

11 **a** $f_x = \frac{1}{x}$

b $f_y = \frac{1}{y}$

c $f_{xx} = -\frac{1}{x^2}$

d $f_{yy} = -\frac{1}{y^2}$

Proof

12 **a** $f_x = 2xy^2\ln(xy) + xy^2$

b $f_y = 2x^2y\ln(xy) + x^2y$

c $f_{xx} = 2y^2\ln(xy) + 3y^2$

d $f_{yy} = 2x^2\ln(xy) + 3x^2$

Proof

13 **a** $f_x = -\sin x$

b $f_y = -\sin y$

c $f_{xx} = -\cos x$

d $f_{yy} = -\cos y$

Proof

14 **a** $f_x = -\sin x\sin y$

b $f_y = \cos x\cos y$

c $f_{xx} = -\cos x\sin y$

d $f_{yy} = -\cos x\sin y$

Proof

15 **a** $f_x = \cos x e^{\sin x - \cos y}$

b $f_y = \sin y e^{\sin x - \cos y}$

c $f_{xx} = (\cos^2 x - \sin x)e^{\sin x - \cos y}$

d $f_{yy} = (\sin^2 y + \cos y)e^{\sin x - \cos y}$

Proof

16 $\frac{\partial^2 z}{\partial x^2} = 3, \frac{\partial^2 z}{\partial y^2} = 5, \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = 2.$ Proof

17 Proof

18 Proof

19 Proof

20 a $f_x = 3x^2 + 6xy + 3z^2 + 2xy^2z^2$

b $f_y = 3y^2 + 3x^2 + 6yz + 2x^2yz^2$

c $f_z = 3z^2 + 3y^2 + 6xz + 2x^2y^2z$

d $f_{xyz} = 8xyz$

Exercise 5C

1 $(0, 0, 0)$ proof

2 $(0, 0, 4)$ max

3 Proof

4 $(2, 2, -8)$ min, $(0, 0, 0)$ saddle

5 $(0, 0, 0)$ max, $(0, 4, -32)$ min, $(\pm\sqrt{3}, 1, -5)$ saddle

6 Proof

$(-1, 0, -1)$ min, $(1, 0, -1)$ min,

$(0, 0, 0)$ saddle

7 Proof

$(\pm 1, \pm 1, 1)$ saddle, $(0, 0, 0)$ min

8 a $(-1, -1, 9)$ max, $(2, 1, -22)$ min

b $(2, -1, -18)$ and $(-1, 1, 5)$ saddle

9 Proof

$(-1, 0.5, -6)$ and $(1, -0.5, -4)$ saddle

10 Proof

$(0, 0, 0)$ max,

$(\pm 1, \pm 1, -2)$ min,

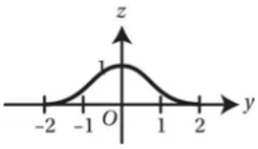
$(\pm 1, 0, -1)(0, \pm 1, -1)$ saddle

11 $(0, 0, 1)$, proof

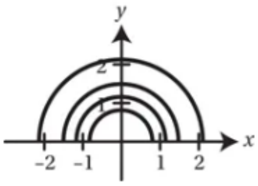
12 Proof

13 a Proof

b



c



14 Proof

$\left(0, \frac{\pi}{2}\right)$ max, $\left(\pi, \frac{\pi}{2}\right)$ saddle

15 $(-1, 1, -1 + \ln 2)$ saddle

$(1, -1, -1 + \ln 2)$ saddle

16 Proof

17 Proof

$(-1, -1, 4)$ max, $(1, 1, -4)$ min, $(1, -1, 0), (-1, 1, 0)$ saddle

18 a Proof

b $\left(-\frac{\pi}{2}, 0\right)$ saddle, $\left(-\frac{\pi}{2}, \pi\right)$ min, $\left(\frac{\pi}{2}, 0\right)$ max, $\left(\frac{\pi}{2}, \pi\right)$ saddle

19 Proof. $(0, 0, 0)$ min if $k^2 < 1$, saddle if $k^2 > 1$

20 Proof

Exercise 5D

1 Proof

2 $z = 4x - 6y + 5$

3 $z = -16x + 46 + 79$

4 Proof

$z = 14x - 2y - 14$

5 Proof

6 $z = -7x + 7y - 2$

7 $z = 13x + 6y - 30$

8 $2z = x - 2y + 4$

9 $4z = 2x - 3y + 14$

10 $4z = 27x + 36y - 117$

11 Proof

12 $z = 4e(2x + y - 3)$

13 Proof

14 $2z = -2y + \pi + 2$

15 $2z = -x + y + 1$

16 Proof

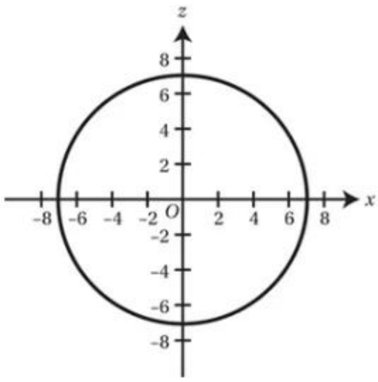
17 Proof

18 Yes, parallel to $z = x + y - 1$

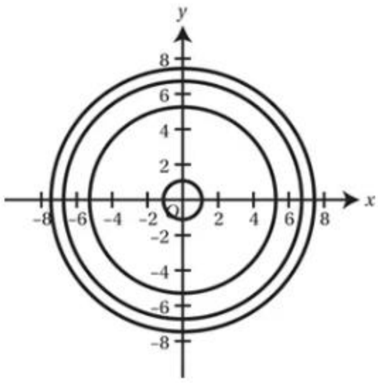
19 $z = x$

20 a Proof

b

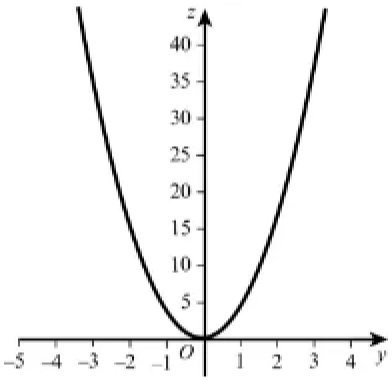


c

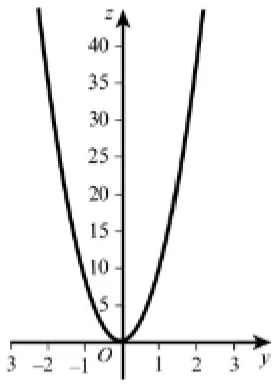


Mixed practice 5

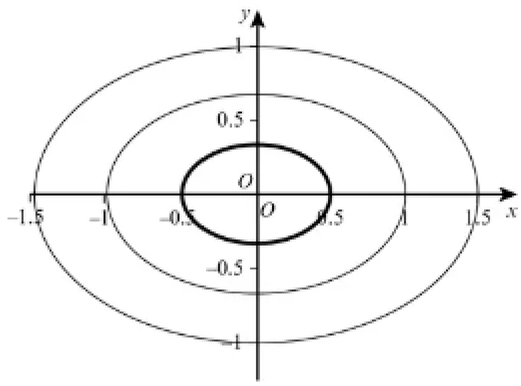
1 a i



ii



b



2 **a** Proof

b Proof

3 $2z = 19 - 6x - y$

4 Proof

5 Proof

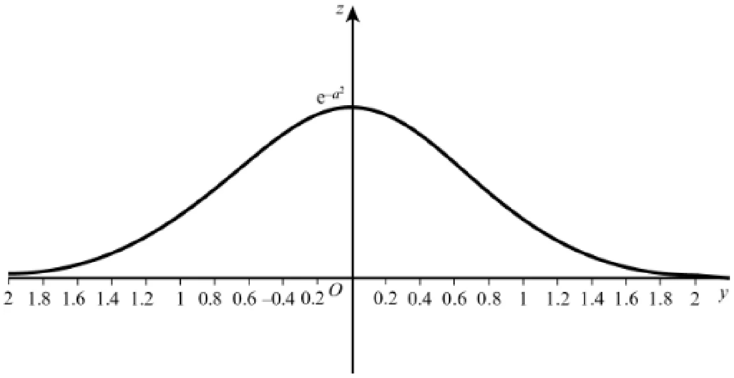
6 $z = x - 10y + 10$

7 **a** Proof

b $(0, 0, 0)$, maximum since

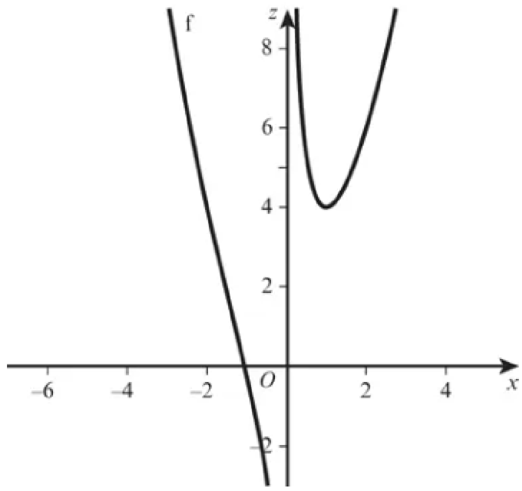
$f_{xx} = -2 < 0, f_{yy} = -2 < 0, f_{xx}f_{yy} - {f_{xy}}^2 = 4 > 0$

c $(a, 0, \mathrm{e}^{-a^2})$



8 **a** $(1, 1, 4)$ min, $(-1, -1, 4)$ min

b



9 **a** $A = xy + \frac{2}{x} + \frac{2}{y}$

b Proof

10 $5x + 4y + 3z = 22$

11 $\frac{\partial u}{\partial x} = \frac{v^2 + 2uvx}{2uy^2 - 2vx^2}$, proof