Matrices – Post 2018 Mix

1.

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

(a) Show that 2 is a repeated eigenvalue of $\mathbf{A}$ and find the other eigenvalue.

(5)

(b) Hence find three non-parallel eigenvectors of $\mathbf{A}$.

(4)

(c) Find a matrix $\mathbf{P}$ such that $\mathbf{P}^{-1}\mathbf{AP}$ is a diagonal matrix.

(2)

2.

$$\mathbf{M} = \begin{pmatrix} 1 & k & -2 \\ 2 & -4 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

where k is a constant.

(a) Show that, in terms of k, a characteristic equation for $\mathbf{M}$ is given by

$$\lambda^3 - (2k + 13)\lambda + 5(k + 6) = 0$$

(3)

Given that det $\mathbf{M} = 5$

(b) (i) find the value of k

(ii) use the Cayley-Hamilton theorem to find the inverse of $\mathbf{M}$.

(7)

3.

$$\mathbf{A} = \begin{pmatrix} 5 & -2 & 5 \\ 0 & 3 & p \\ -6 & 6 & -4 \end{pmatrix} \quad \text{where } p \text{ is a constant}$$

Given that $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ is an eigenvector for **A**

(a) (i) determine the eigenvalue corresponding to this eigenvector

(1)

(ii) hence show that $p = 2$

(2)

(iii) determine the remaining eigenvalues and corresponding eigenvectors of **A**

(7)

(b) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{-1}$

(1)

4.

Matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & a \\ -3 & b & 1 \\ 0 & 1 & a \end{pmatrix}$$

where a and b are integers, such that $a < b$

Given that the characteristic equation for **M** is

$$\lambda^3 - 7\lambda^2 + 13\lambda + c = 0$$

where c is a constant,

(a) determine the values of a, b and c.

(5)

(b) Hence, using the Cayley–Hamilton theorem, determine the matrix $\mathbf{M}^{-1}$

(3)

5.

$$\mathbf{A} = \begin{pmatrix} -1 & a \\ 3 & 8 \end{pmatrix}$$

where a is a constant.

(a) Determine, in expanded form in terms of a, the characteristic equation for $\mathbf{A}$.

(2)

(b) Hence use the Cayley-Hamilton theorem to determine values of a and b such that

$$\mathbf{A}^3 = \mathbf{A} + b\mathbf{I}$$

where $\mathbf{I}$ is the 2×2 identity matrix.

(4)

6.

The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}$$

(a) Show that 4 is an eigenvalue of **M**, and find the other two eigenvalues.

(4)

(b) For each of the eigenvalues find a corresponding eigenvector.

(4)

(c) Find a matrix **P** such that $\mathbf{P}^{-1}\mathbf{M}\mathbf{P}$ is a diagonal matrix.

(2)