The Knowledge Self-Check

Discriminant and how it determines the number of roots of a quadratic

$b^2 - 4ac$, if >0 then 2 distinct real roots, =0 then one repeated real root, <0 no real roots

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The factor theorem

(x-a) is a factor of f(x) if and only if f(a)=0

Sine rule, cosine rule and area of a triangle

$\frac{sin\,A}{a} = \frac{sin\,B}{b}$, $a^2 = b^2 + c^2 - 2bc\,cos\,A$, $\text{Area} = \frac{1}{2}ab\,sin\,C$

Equation of a circle

Circle with centre (a,b) and radius *r*: $(x - a)^2 + (y - b)^2 = r^2$

Graphs of sin, cos and tan (in both degrees and radians)

Check on Desmos

Special values of sin, cos and tan (in both degrees and radians)

Check with your calculator – should know for $0°, 30°, 45°, 60°, 90°$

Trigonometric identity for $\tan\theta$, the "Pythagorean" identity, and the two that derive from this

$tan\,\theta = \frac{sin\,\theta}{cos\,\theta}$, $sin^2\,\theta + cos^2\,\theta = 1$, $tan^2\,\theta + 1 = sec^2\,\theta$, $1 + cot^2\,\theta = \text{cosec}^2\theta$

Laws of logarithms

$log\,A + log\,B = log(AB)$, $log\,A - log\,B = log\left(\frac{A}{B}\right)$, $log\,A^n = n\,log\,A$

Graphs of $y = x^2$, $y = x^3$, $y = \frac{1}{x}$, $y = \sqrt{x}$, $y = a^x$, $y = log_a\,x$

Check with Desmos

How to determine the nature of stationary points from $\frac{d^2y}{dx^2}$

If >0 then local min, <0 then local max, =0 need to check gradient either side

Definitions of concave upwards and concave downwards

Concave upwards means $\frac{d^2y}{dx^2} > 0$, concave downwards means $\frac{d^2y}{dx^2} < 0$

Derivatives of $y = x^n$ and $y = e^{kx}$

Function	***Differentiated***
$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$
$y = e^{kx}$	$\frac{dy}{dx} = ke^{kx}$

Graph transformations$y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$, $y = -f(x)$, $y = f(-x)$

Translation by $\binom{0}{a}$, translation by $\binom{-a}{0}$, stretch parallel to *y*-axis by sf a, stretch parallel to *x*-axis by sf $\frac{1}{a}$, reflection in *x*-axis, reflection in *y*-axis

The 5 constant acceleration formulae (SUVAT)

$v = u + at$, $s = vt - \frac{1}{2}at^2$, $s = ut + \frac{1}{2}at^2$, $s = \frac{u+v}{2}t$, $v^2 = u^2 + 2as$

Special sums

$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$, $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$

$|wz|$ and $arg(wz)$ in terms of $|w|$, $|z|$, $arg(w)$, $arg(z)$

$|wz| = |w||z|$, $arg(wz) = arg(w) + arg(z)$

Modulus-argument form for a complex number with modulus r and argument θ

$r(\cos\theta + i\sin\theta)$

Complex loci for the forms $|z - \alpha| = r$, $\arg(z - \alpha) = \theta$ and $|z - \alpha| = |z - \beta|$

Circle radius r centred at α, half-line starting at α and at an angle of θ above the positive horizontal, perpendicular bisector of α and β

Arc length and area of sector formulae given angle in radians

$\text{Arc length} = \theta r$, $\text{Area of sector} = \frac{\theta r^2}{2}$

nth terms and partial sums of arithmetic and geometric sequences (and sum to infinity)

Arithmetic: $u_n = a + (n - 1)d$, $S_n = \frac{n}{2}[2a + (n - 1)d]$

Geometric: $u_n = ar^{n-1}$, $S_n = \frac{a(r^n-1)}{r-1}$, $S_\infty = \frac{a}{1-r}$ (if $|r| < 1$)

Small angle approximations for $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{\theta^2}{2}$, $\tan\theta \approx \theta$, where θ is in radians

Standard matrix transformations in 2D

Reflection in *x*-axis $\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$, reflection in *y*-axis $\begin{pmatrix}-1 & 0\\0 & 1\end{pmatrix}$

Reflection in $y = x$ $\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$, reflection in $y = -x$ $\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$

Rotation through θ anti-clockwise about O $\begin{pmatrix}\cos\theta & -\sin\theta\\\sin\theta & \cos\theta\end{pmatrix}$

Stretch s.f. *c* parallel to x-axis, s.f. *d* parallel to *y*-axis $\begin{pmatrix}c & 0\\0 & d\end{pmatrix}$

Shear, *x*-axis fixed, with $(0,1)$ mapped to $(k,1)$: $\begin{pmatrix}1 & k\\0 & 1\end{pmatrix}$,

Shear, y-axis fixed, with $(1,0)$ mapped to $(1,k)$: $\begin{pmatrix}1 & 0\\k & 1\end{pmatrix}$

Inverse and determinant of 2x2 matrices, and geometric significance of determinant

$\begin{vmatrix}a & b\\c & d\end{vmatrix} = ad - bc$, if $M = \begin{pmatrix}a & b\\c & d\end{pmatrix}$ then $M^{-1} = \frac{1}{ad-bc}\begin{pmatrix}d & -b\\-c & a\end{pmatrix}$, $|det\, M|$ =area scale factor

Roots of polynomials for quadratics, cubics, quartics (sums, products etc.)

If α, β are roots of $ax^2 + bx + c = 0$ then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$

If α, β, γ are roots of $ax^3 + bx^2 + cx + d = 0$ then $\alpha + \beta + \gamma = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$, $\alpha\beta\gamma = -\frac{d}{a}$

If $\alpha, \beta, \gamma, \delta$ are roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$ then $\alpha + \beta + \gamma + \delta = -\frac{b}{a}$, $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$, $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$, $\alpha\beta\gamma\delta = \frac{e}{a}$

Double angle - $sin(2\theta)$, $cos(2\theta)$ (in 3 different ways) and $tan(2\theta)$

$\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$, $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

Standard matrix transformations in 3D

Reflection in the plane $x = 0$ $\begin{pmatrix}-1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{pmatrix}$, reflection in the plane $y = 0$ $\begin{pmatrix}1 & 0 & 0\\0 & -1 & 0\\0 & 0 & 1\end{pmatrix}$, reflection in the plane $z = 0$ $\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{pmatrix}$.

Rotation through θ about the x-axis $\begin{pmatrix}1 & 0 & 0\\0 & \cos\theta & -\sin\theta\\0 & \sin\theta & \cos\theta\end{pmatrix}$, rotation through θ about the y-axis $\begin{pmatrix}\cos\theta & 0 & \sin\theta\\0 & 1 & 0\\-\sin\theta & 0 & \cos\theta\end{pmatrix}$, rotation through θ about the z-axis $\begin{pmatrix}\cos\theta & -\sin\theta & 0\\\sin\theta & \cos\theta & 0\\0 & 0 & 1\end{pmatrix}$.

Product and quotient rules

If $y = uv$, $\frac{dy}{dx} = u'v + v'u$. If $y = \frac{u}{v}$, $\frac{dy}{dx} = \frac{u'v - v'u}{v^2}$.

The formula for the angle between two vectors

$$\cos\theta = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|}$$

The general equation of a plane

$\boldsymbol{r} \cdot \boldsymbol{n} = p$, where **n** is the normal to the plane and p is a constant

Chain rule for $y = (\text{stuff})^n$, $y = e^{\text{stuff}}$, $y = a^{\text{stuff}}$, $y = \ln(\text{stuff})$, $y = \sin(\text{stuff})$, $y = \cos(\text{stuff})$, $y = \tan(\text{stuff})$

Simple function	***Differentiated***	***Chain function***	***Differentiated***
$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$	$y = (\text{stuff})^n$	$\frac{dy}{dx} = \boldsymbol{stuff}' \times n(\text{stuff})^{n-1}$
$y = e^x$	$\frac{dy}{dx} = e^x$	$y = e^{\text{stuff}}$	$\frac{dy}{dx} = \boldsymbol{stuff}' \times e^{\text{stuff}}$
$y = a^x$	$\frac{dy}{dx} = (\ln a) \times a^x$	$y = a^{\text{stuff}}$	$\frac{dy}{dx} = \boldsymbol{stuff}' \times (\ln a) \times a^{\text{stuff}}$
$y = \ln(x)$	$\frac{dy}{dx} = \frac{1}{x}$	$y = \ln(\text{stuff})$	$\frac{dy}{dx} = \boldsymbol{stuff}' \times \frac{1}{\text{stuff}}$
$y = \sin x$	$\frac{dy}{dx} = \cos x$	$y = \sin(\text{stuff})$	$\frac{dy}{dx} = \boldsymbol{stuff}' \times \cos(\text{stuff})$
$y = \cos x$	$\frac{dy}{dx} = -\sin x$	$y = \cos(\text{stuff})$	$\frac{dy}{dx} = \boldsymbol{stuff}' \times -\sin(\text{stuff})$
$y = \tan x$	$\frac{dy}{dx} = \sec^2 x$	$y = \tan(\text{stuff})$	$\frac{dy}{dx} = \boldsymbol{stuff}' \times \sec^2(\text{stuff})$

The form of partial fractions for $\frac{mx+n}{(x+p)(x+q)}$, $\frac{mx+n}{(x+p)(x+q)^2}$ and $\frac{mx+n}{(x+p)(x^2+q^2)}$

$\frac{mx+n}{(x+p)(x+q)} = \frac{A}{x+p} + \frac{B}{x+q}$, $\frac{mx+n}{(x+p)(x+q)^2} = \frac{A}{x+p} + \frac{B}{x+q} + \frac{C}{(x+q)^2}$ and $\frac{mx+n}{(x+p)(x^2+q^2)} = \frac{A}{x+p} + \frac{Bx+C}{x^2+q^2}$

The gradient of a curve defined parametrically

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)}$$

The formula for $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Two ways of checking for the independence of events A and B

Either check $P(A \cap B) = P(A) \times P(B)$ or $P(A|B) = P(A)$

Both definitions of statistical outliers

More than 2 standard deviations away from the mean or more than 1.5 IQR away from the nearest quartile

Distribution of the sample mean of *n* Normally distributed random variables sampled from $N(\mu, \sigma^2)$

$$\bar{X}_n \sim N\left(\mu, \frac{\sigma^2}{n}\right)$$