

The Knowledge Self-Check

Discriminant and how it determines the number of roots of a quadratic

$b^2 - 4ac$, if >0 then 2 distinct real roots, $=0$ then one repeated real root, <0 no real roots

Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The factor theorem

$(x-a)$ is a factor of $f(x)$ if and only if $f(a)=0$

Sine rule, cosine rule and area of a triangle

$$\frac{\sin A}{a} = \frac{\sin B}{b}, a^2 = b^2 + c^2 - 2bc \cos A, \text{Area} = \frac{1}{2}ab \sin C$$

Equation of a circle

Circle with centre (a,b) and radius r : $(x - a)^2 + (y - b)^2 = r^2$

Graphs of sin, cos and tan (in both degrees and radians)

Check on Desmos

Special values of sin, cos and tan (in both degrees and radians)

Check with your calculator – should know for $0^\circ, 30^\circ, 45^\circ, 60^\circ, 90^\circ$

Trigonometric identity for $\tan\theta$, the “Pythagorean” identity, and the two that derive from this

$$\tan \theta = \frac{\sin \theta}{\cos \theta}, \sin^2 \theta + \cos^2 \theta = 1, \tan^2 \theta + 1 = \sec^2 \theta, 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

Laws of logarithms

$$\log A + \log B = \log(AB), \log A - \log B = \log\left(\frac{A}{B}\right), \log A^n = n \log A$$

Graphs of $y = x^2$, $y = x^3$, $y = \frac{1}{x}$, $y = \sqrt{x}$, $y = a^x$, $y = \log_a x$

Check with Desmos

How to determine the nature of stationary points from $\frac{d^2y}{dx^2}$

If >0 then local min, <0 then local max, $=0$ need to check gradient either side

Definitions of concave upwards and concave downwards

Concave upwards means $\frac{d^2y}{dx^2} > 0$, concave downwards means $\frac{d^2y}{dx^2} < 0$

Derivatives of $y = x^n$ and $y = e^{kx}$

Function	Differentiated
$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$
$y = e^{kx}$	$\frac{dy}{dx} = ke^{kx}$

Graph transformations $y = f(x) + a$, $y = f(x + a)$, $y = af(x)$, $y = f(ax)$, $y = -f(x)$, $y = f(-x)$

Translation by $\begin{pmatrix} 0 \\ a \end{pmatrix}$, translation by $\begin{pmatrix} -a \\ 0 \end{pmatrix}$, stretch parallel to y -axis by sf a , stretch parallel to x -axis by sf $\frac{1}{a}$, reflection in x -axis, reflection in y -axis

The 5 constant acceleration formulae (SUVAT)

$$v = u + at, s = vt - \frac{1}{2}at^2, s = ut + \frac{1}{2}at^2, s = \frac{u+v}{2}t, v^2 = u^2 + 2as$$

Special sums

$$\sum_{r=1}^n r = \frac{n(n+1)}{2}, \sum_{r=1}^n r^2 = \frac{n(n+1)(2n+1)}{6}, \sum_{r=1}^n r^3 = \frac{n^2(n+1)^2}{4}$$

$|wz|$ and $\arg(wz)$ in terms of $|w|$, $|z|$, $\arg(w)$, $\arg(z)$

$$|wz| = |w||z|, \arg(wz) = \arg(w) + \arg(z)$$

Modulus-argument form for a complex number with modulus r and argument θ

$$r(\cos \theta + i \sin \theta)$$

Complex loci for the forms $|z - \alpha| = r$, $\arg(z - \alpha) = \theta$ and $|z - \alpha| = |z - \beta|$

Circle radius r centred at α , half-line starting at α and at an angle of θ above the positive horizontal, perpendicular bisector of α and β

Arc length and area of sector formulae given angle in radians

$$\text{Arc length} = \theta r, \text{Area of sector} = \frac{\theta r^2}{2}$$

nth terms and partial sums of arithmetic and geometric sequences (and sum to infinity)

$$\text{Arithmetic: } u_n = a + (n-1)d, S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{Geometric: } u_n = ar^{n-1}, S_n = \frac{a(r^n-1)}{r-1}, S_\infty = \frac{a}{1-r} \text{ (if } |r| < 1\text{)}$$

Small angle approximations for $y = \sin \theta$, $y = \cos \theta$ and $y = \tan \theta$

$$\sin \theta \approx \theta, \cos \theta \approx 1 - \frac{\theta^2}{2}, \tan \theta \approx \theta, \text{ where } \theta \text{ is in radians}$$

Standard matrix transformations

Reflection in x-axis $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, reflection in y-axis $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

Reflection in $y = x$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, reflection in $y = -x$ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

Rotation through θ anti-clockwise about O $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

Stretch s.f. c parallel to x-axis, s.f. d parallel to y-axis $\begin{pmatrix} c & 0 \\ 0 & d \end{pmatrix}$

Shear, x-axis fixed, with $(0,1)$ mapped to $(k, 1)$: $\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$,

Shear, y-axis fixed, with $(1,0)$ mapped to $(1, k)$: $\begin{pmatrix} 1 & 0 \\ k & 1 \end{pmatrix}$

Inverse and determinant of 2x2 matrices, and geometric significance of determinant

$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$, if $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ then $M^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, $|det M|$ = area scale factor

Roots of polynomials for quadratics, cubics, quartics (sums, products etc.)

If α, β are roots of $ax^2 + bx + c = 0$ then $\alpha + \beta = -\frac{b}{a}$, $\alpha\beta = \frac{c}{a}$

If α, β, γ are roots of $ax^3 + bx^2 + cx + d = 0$ then $\alpha + \beta + \gamma = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$,
 $\alpha\beta\gamma = -\frac{d}{a}$

If $\alpha, \beta, \gamma, \delta$ are roots of $ax^4 + bx^3 + cx^2 + dx + e = 0$ then $\alpha + \beta + \gamma + \delta = -\frac{b}{a}$, $\alpha\beta + \alpha\gamma + \alpha\delta + \beta\gamma + \beta\delta + \gamma\delta = \frac{c}{a}$, $\alpha\beta\gamma + \alpha\beta\delta + \alpha\gamma\delta + \beta\gamma\delta = -\frac{d}{a}$, $\alpha\beta\gamma\delta = \frac{e}{a}$

Double angle formulae - $\sin(2\theta)$, $\cos(2\theta)$ (in 3 different ways) and $\tan(2\theta)$

$\sin 2\theta = 2 \sin \theta \cos \theta$, $\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$, $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$